# Undirected graphs

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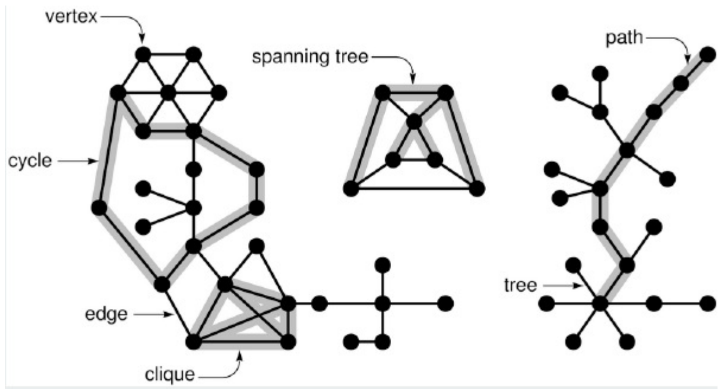
## Undirected graphs

- A graph G=(V, E) where V is a set of vertices connected pairwise by edges E.
- Why study graph algorithms?
  - Interesting and broadly useful abstraction.
  - Challenging branch of computer science and discrete math.
  - Hundreds of graph algorithms known.
  - Thousands of practical applications.
    - Communication, circuits, transportation, scheduling, software systems, internet, games, social network, neural networks, …

## Graph terminology

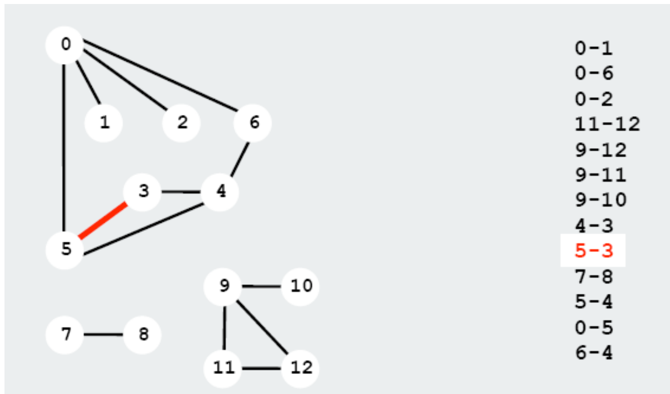



## Some graph-processing problems

- Path: Is there a path between s to t?
- Shortest path: What is the shortest path between s and t?
- Cycle: Is there a cycle in the graph?
- Euler tour: Is there a cycle that uses each edge exactly once?
- Hamilton tour: Is there a cycle that uses each vertex exactly once?
- Connectivity: Is there a way to connect all of the vertices?
- MST: What is the best way to connect all of the vertices?
- Biconnectivity: Is there a vertex whose removal disconnects the graph?

## Graph representation (1)

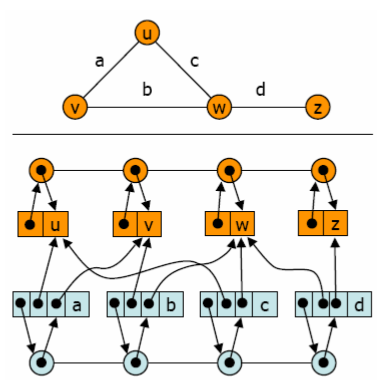
- Maintain a list of the edges




- Not suitable for searching

## Edge List Structure

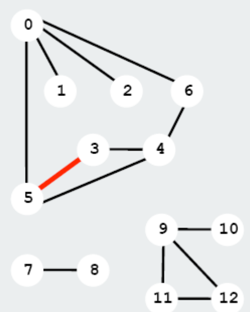
- Edge sequence
  - sequence of edge objects
- Edge object
  - element
  - origin vertex object
  - destination vertex object
  - reference to position in edge sequence
- Vertex sequence
  - sequence of vertex objects
- Vertex object
  - – element
  - – reference to position in vertex sequence



## Graph representation (2)

- Maintain an adjacency matrix.

For each edge v-w in graph: `adj[v][w] = adj[w][v] = true.`




two entries for each edge

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |

- Suitable for random accesses to the edges

## A graph data structure

- Use a dynamic array to represent a graph as the following

```
typedef struct {
    int * matrix;
    int sizemax;
} Graph;
```

- Define the following API

```
Graph createGraph(int sizemax);
void setEdge(Graph* graph, int v1, int v2);
int connected(Graph* graph, int v1, int v2);
int getConnectedVertices(Graph* graph, int vertex, int[]
    output); // return the number of connected vertices.
```

## How to use the API?

```
int i, n, output[100];
Graph g = createGraph(100);
addEdge(g, 0, 1);
addEdge(g, 0, 2);
addEdge(g, 1, 2);
addEdge(g, 1, 3);
n = getAdjacentVertices (g, 1, output);
if (n==0) printf("No adjacent vertices of node 1\n");
else {
  printf("Adjacent vertices of node 1:");
  for (i=0; i<n; i++) printf("%5d", output[i]);
}
```

## Quiz 1

- Write the implementation for the API defined in the previous slide
- Use the example to test your API

## Quiz 2

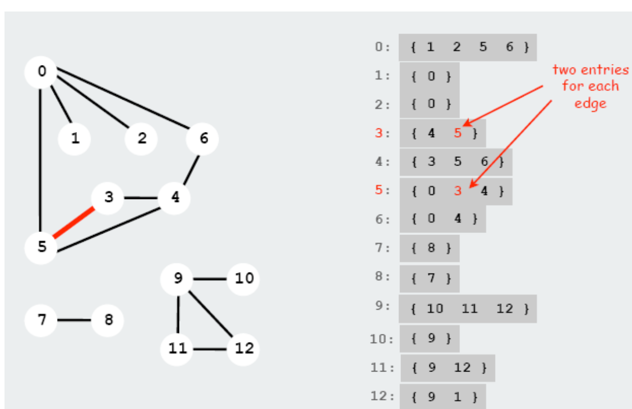
- In order to describe the metro lines of a city, we can store the data in a file as the following.

```
[STATIONS]
S1=Name of station 1
S2=Name of station 2
…
[LINES]
M1=S1 S2 S4 S3 S7
M2=S3 S5 S6 S8 S9
…
```

- Make a program to read such a file and establish the network of metro stations in the memory using a two-dimensional array.
- Write a function to find all the stations directly connected to a station given by its name.

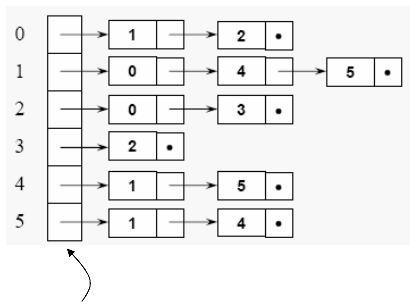
## Graph representation (3)

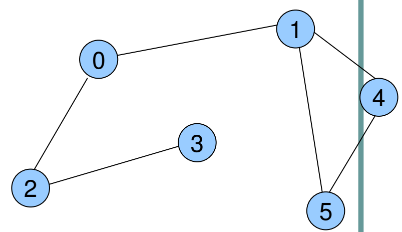
- Maintain an adjacency list.



## Adjacency List Representation

- A graph may also be represented by an adjacency list structure:






**Array of linked lists, where list nodes store node labels for neighbors.**

## Implementation

- The red black tree can be used to store such a graph where each node in the tree is a vertex and its value is a set of connected vertices.
- The set of connected vertices is stored in a red black tree it self.

## Quiz 2

- Reuse the libfdr library to implement an API for manipulating the graph as the following

```
typedef JRB Graph;
Graph createGraph();
void setEdge(Graph* graph, int v1, int v2);
int connected(Graph* graph, int v1, int v2);
void forEachConnectedVertex(Graph* graph, int vertex, void
    (*func)(int, int) );
// the last one is a navigation function that iterates over all
    connected vertices of a given to do something. func is a
    pointer to the function that process on the connected vertices.
```

- Rewrite the metro network program using this new API

## Comparison

- Adjacency List is usually preferred, because it provides a compact way to represent sparse graphs – those for which |E| is much less than $|V|^2$
- Adjacency Matrix may be preferred when the graph is dense, or when we need to be able to tell quickly if their is an edge connecting two given vertices

## Quiz 3

- Rewrite the API defined for graphs using the libfdr library as the following

```
#include "jrb.h"
typedef JRB Graph;

Graph createGraph();
void addEdge(Graph graph, int v1, int v2);
int adjacent(Graph graph, int v1, int v2);
int getAdjacentVertices (Graph graph, int v, int*
   output);
void dropGraph(Graph graph);
```

## Instructions (1)

- To create a graph

  Simply call make_jrb()
- To add a new edge (v1, v2) to graph g

```
tree = make_jrb();
jrb_insert_int(g, v1, new_jval_v(tree));
jrb_insert_int(tree, v2, new_jval_i(1));
```

- If the node v1 is already allocated in the graph

```
node = jrb_find_int(g, v1);
tree = (JRB) jval_v(node->val);
jrb_insert_int(tree, v2, new_jval_i(1));
```

## Instructions (2)

- To get adjacent vertices of v in graph g

```
node = jrb_find_int(g, v);
tree = (JRB) jval_v(node->val);
total = 0;
jrb_traverse(node, tree)
    output[total++] = jval_i(node->key);
```

- To delete/free a graph

```
jrb_traverse(node, graph)
    jrb_free_tree( jval_v(node->val) );
```

## Solution

- graph_jrb.c

## Homework

- Redo the quiz 2 using the Graph library you have created in quiz 3