Red-black trees

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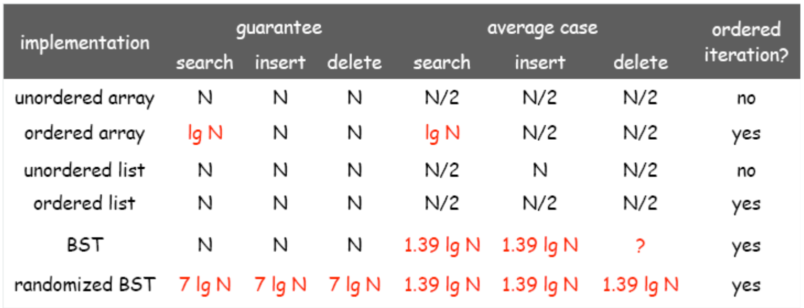
Symbol Table Review

Symbol table: key-value pair abstraction.

- Insert a value with specified key.
- Search for value given key.
- Delete value with given key.

- Different implementations
  - Array
  - Linked list
  - BST (binary search tree)

Complexity

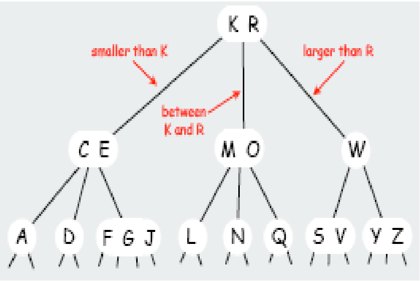


implementation	guarantee			average case			ordered iteration?
	search	insert	delete	search	insert	delete	
unordered array	N	N	N	N/2	N/2	N/2	no
ordered array	lg N	N	N	lg N	N/2	N/2	yes
unordered list	N	N	N	N/2	N	N/2	no
ordered list	N	N	N	N/2	N/2	N/2	yes
BST	N	N	N	1.39 lg N	1.39 lg N	?	yes
randomized BST	7 lg N	7 lg N	7 lg N	1.39 lg N	1.39 lg N	1.39 lg N	yes

- Randomized BST.
  - Guarantee of ~c lg N time per operation (probabilistic).
  - Need subtree count in each node.
  - Need random numbers for each insert/delete op.

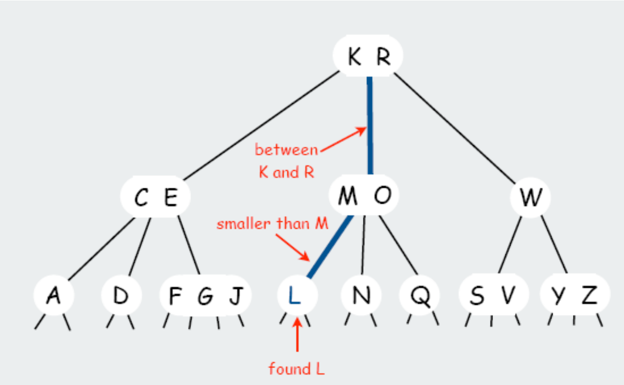
2-3-4 tree

- 2-3-4 tree. Generalize node to allow multiple keys; help to keep tree balanced.
- Perfect balance. Every path from root to leaf has same length.
- Allow 1, 2, or 3 keys per node.
  - 2-node: one key, two children.
  - 3-node: two keys, three children.
  - 4-node: three keys, four children.



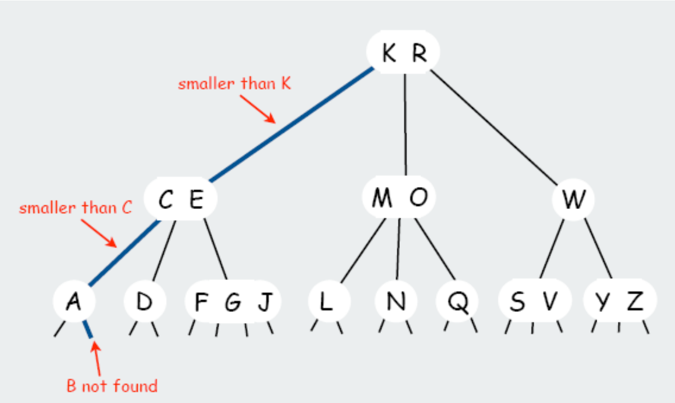
Search

- Compare search key against keys in node.
- Find interval containing search key.
- Ex. Search for L



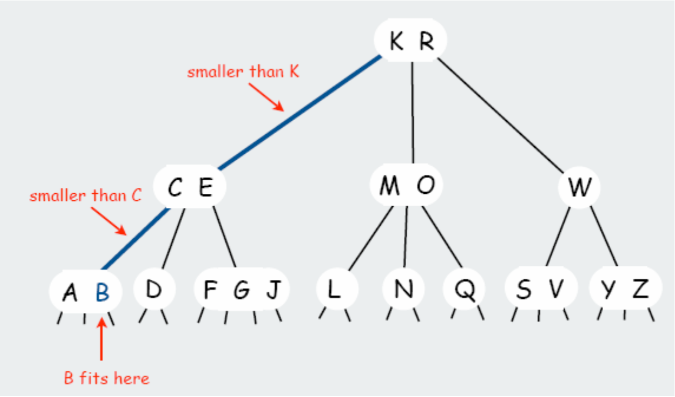
Insert (1)

- Search to bottom for key.
- Ex. Insert B



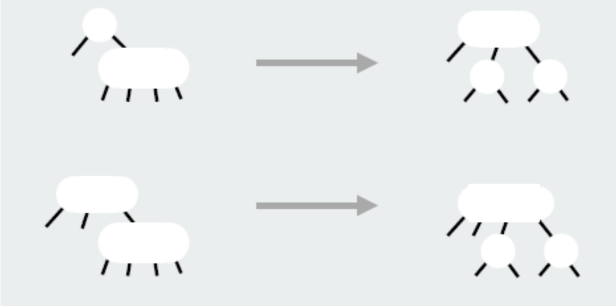
Insert (2)

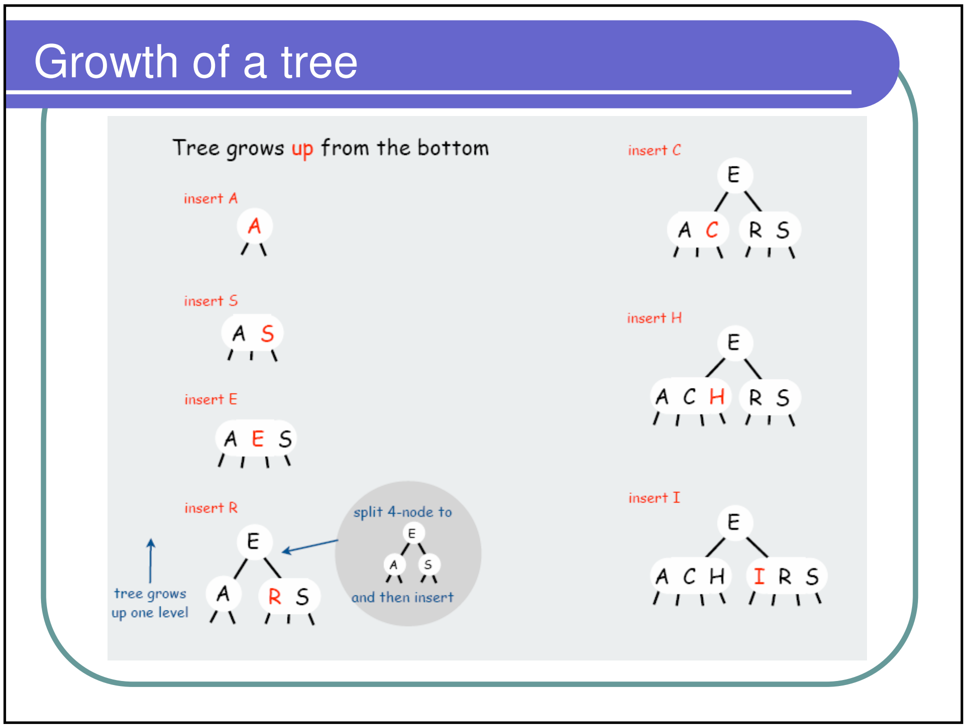
- 2-node at bottom: convert to 3-node.
- 3-node at bottom: convert to 4-node.
- Ex. Insert B



Transformation

- Local transformations should be applied to keep the tree balanced.
- Ensures that most recently seen node is not a 4-node.
- Transformations to split 4-nodes:





Growth of a tree

Tree grows up from the bottom

insert A

insert S

insert E

insert R

tree grows up one level

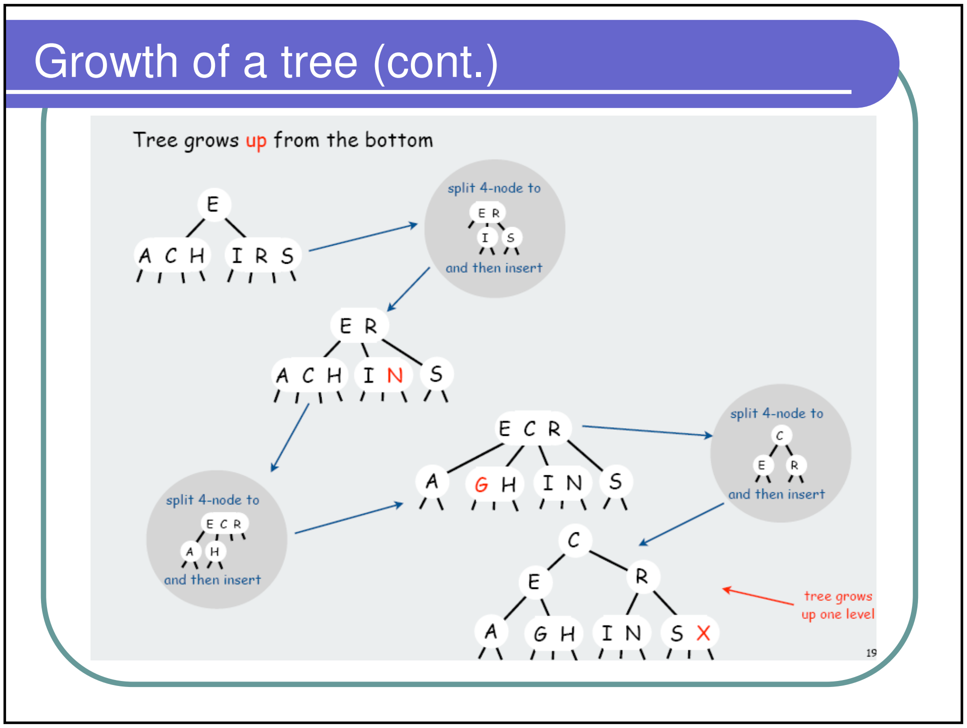
split 4-node to

and then insert

insert C

insert H

insert I



Growth of a tree (cont.)

Tree grows up from the bottom

split 4-node to

and then insert

split 4-node to

and then insert

split 4-node to

and then insert

tree grows up one level

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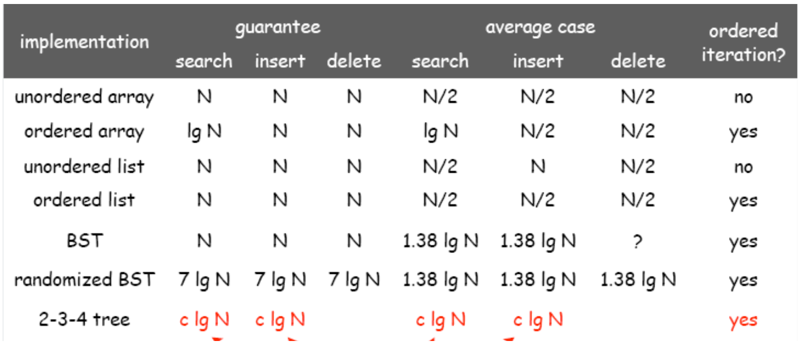
Complexity



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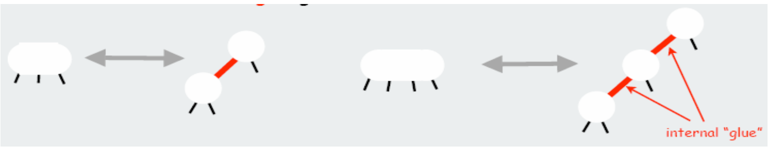
2-3-4 tree	c lg N	c lg N		c lg N	c lg N		yes

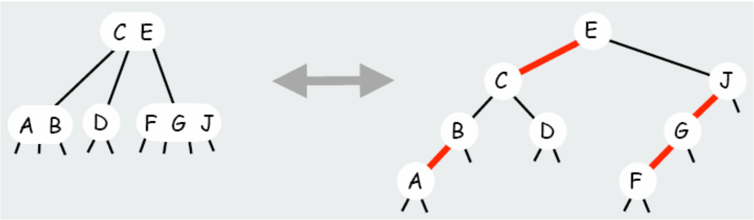
- Tree height
  - Worst case: lg N [all 2-nodes]
  - Best case: log4 N = 1/2 lg N [all 4-nodes]
  - Between 10 and 20 for a million nodes.
  - Between 15 and 30 for a billion nodes.

Red-black tree

- Represent 2-3-4 tree as a BST.
- Use "internal" left-leaning edges for 3- and 4- nodes.

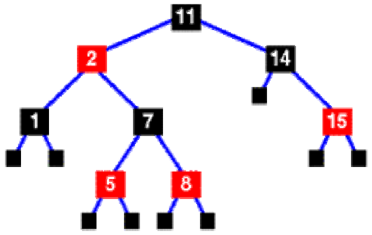


- 1-1 correspondence between 2-3-4 and left-leaning red-black trees.



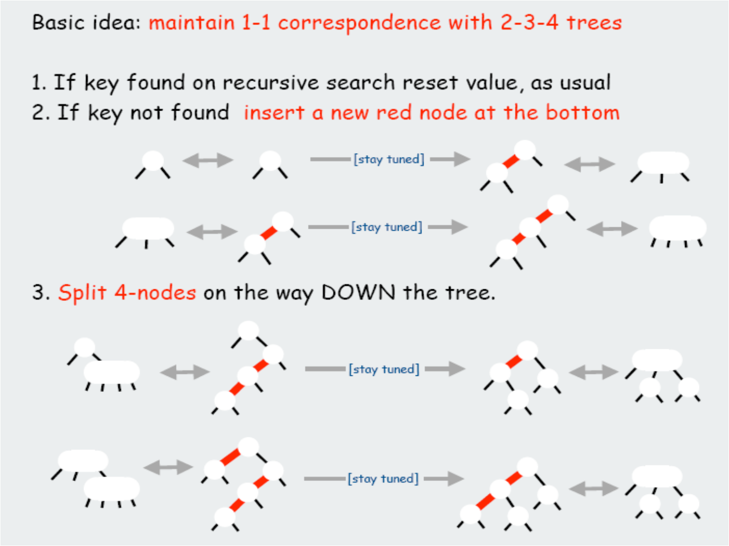
Red-black tree

- A node is either red or black.
- The root is black.
- All leaves are black, even when the parent is black (The leaves are the *null* children.)
- Both children of every red node are black.
- Every simple path from a node to a descendant leaf contains the same number of black nodes



The longest path to a leaf node in the tree will never be more than twice as long as the shortest path

Insert implementation

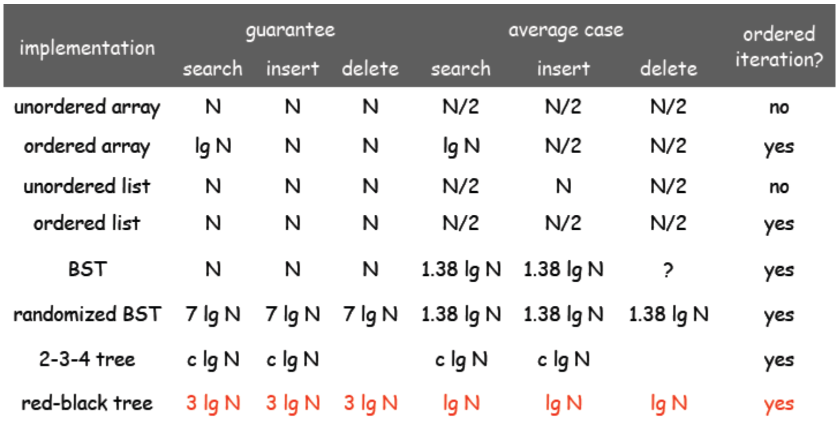


Basic idea: maintain 1-1 correspondence with 2-3-4 trees

1. If key found on recursive search reset value, as usual
2. If key not found insert a new red node at the bottom

3. Split 4-nodes on the way DOWN the tree.

Complexity



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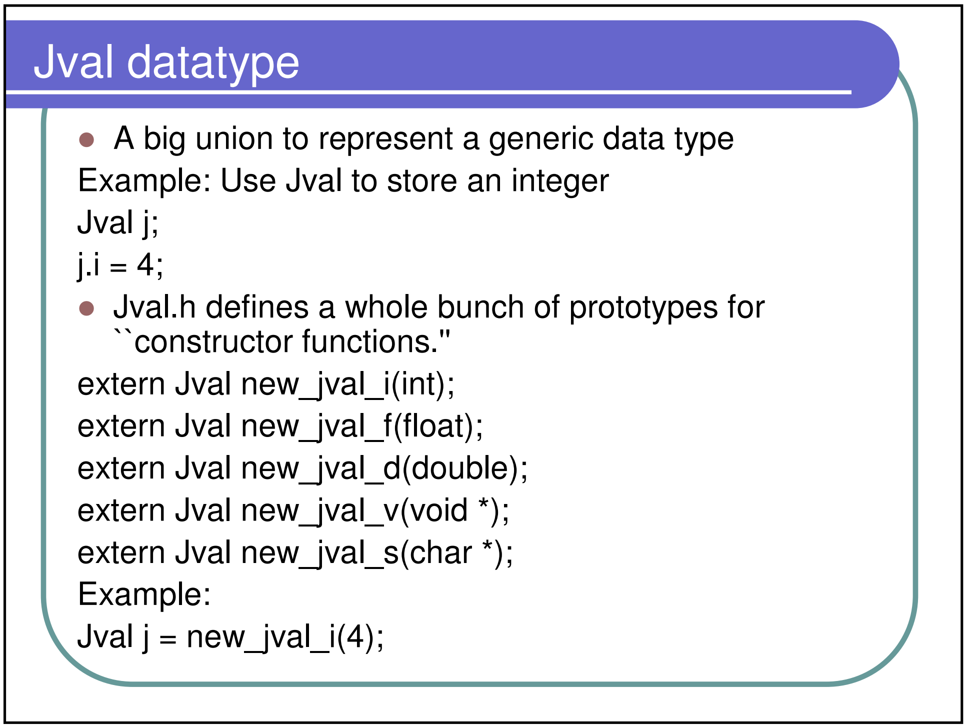
red-black tree	3 lg N	3 lg N	3 lg N	lg N	lg N	lg N	yes

Libfdr

- Libfdr is a library which contains an implementation for generic red-black trees in C
- Download and compile instructions at

http://www.cs.utk.edu/~plank/plank/classes/cs360/360/notes/Libfdr/

http://www.cs.utk.edu/~plank/plank/libfdr/libfdr.tar



Jval datatype

- A big union to represent a generic data type

Example: Use Jval to store an integer

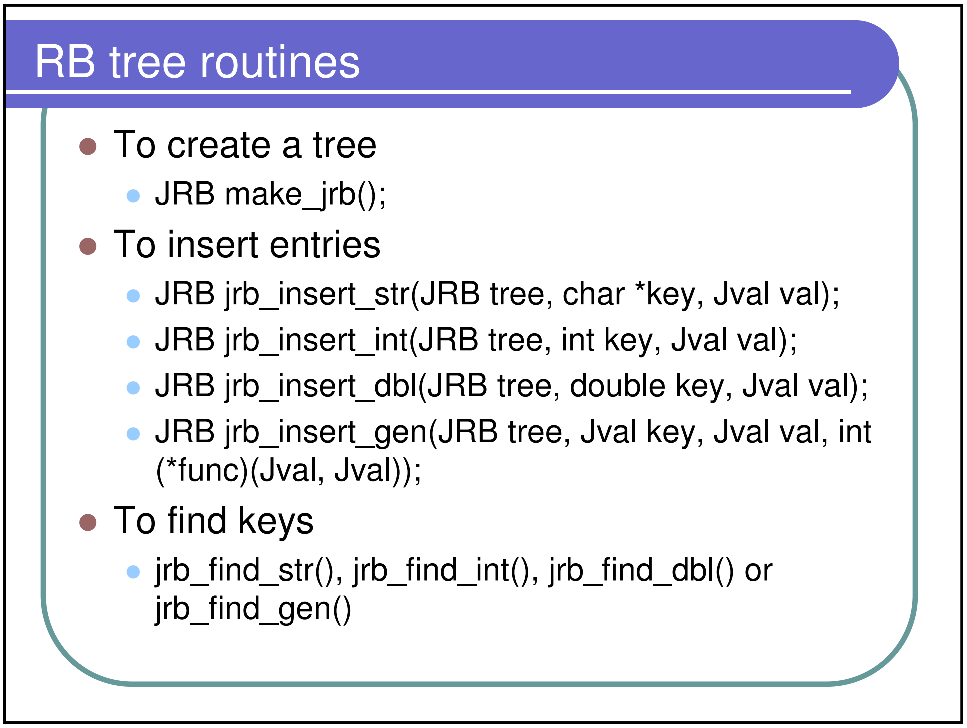
```
Jval j;
j.i = 4;
```

- Jval.h defines a whole bunch of prototypes for ``constructor functions."

```
extern Jval new_jval_i(int);
extern Jval new_jval_f(float);
extern Jval new_jval_d(double);
extern Jval new_jval_v(void *);
extern Jval new_jval_s(char *);
```

Example:

```
Jval j = new_jval_i(4);
```



RB tree routines

- To create a tree
  - JRB make_jrb();
- To insert entries
  - JRB jrb_insert_str(JRB tree, char *key, Jval val);
  - JRB jrb_insert_int(JRB tree, int key, Jval val);
  - JRB jrb_insert_dbl(JRB tree, double key, Jval val);
  - JRB jrb_insert_gen(JRB tree, Jval key, Jval val, int (*func)(Jval, Jval));
- To find keys
  - jrb_find_str(), jrb_find_int(), jrb_find_dbl() or jrb_find_gen()

Quiz 1

- Try to compile and run some example programs (using libfdr) given at

http://www.cs.utk.edu/~plank/plank/classes/cs360/360/notes/JRB/

http://www.cs.utk.edu/~plank/plank/classes/cs360/360/notes/Libfdr/

http://www.cs.utk.edu/~plank/plank/libfdr/libfdr.tar

- You can consult an example makefile at:

http://www.cs.utk.edu/~parker/Courses/CS302-fall05/Notes/Stuff/makefile

http://www.cs.utk.edu/~parker/Courses/CS302-fall05/Notes/Stuff/

Quiz 2

- Use libfdr to write the phone book program (add, delete, insert, modify phone numbers). The phone book should be stored in a file.