Topology Design and Cross-Layer Optimization for FSO Mesh Networks Impaired by Atmospheric Turbulence and Misalignment Fading

Linh D. Truong, Hien T. T. Pham, Ngoc T. Dang, and Toi V. Doan

***Abstract*—In this paper, we design and optimize free-space optics (FSO) mesh networks over atmospheric turbulence and misalignment fading channels. We propose a heuristic algorithm to choose the best sites to install the FSO transceivers and also the best topology for a given traffic matrix in a region. The algorithm aims to use the least of FSO links as possible since this allows to reduce the network costs. In addition, the algorithm takes into account the influence of turbulence, misalignment, and noise by choosing links with low bit-error rate (BER). Beside the heuristic, we proposed also an optimal integer linear programming (ILP) model for solving the same problem. The simulation results show that the proposed heuristic runs very fast, even with a large number of choices of FSO sites. Therefore, it is practical to use the heuristic to design quickly a restoration communication network for replacing a regular one affected by a disaster. The average BER of all FSO links of the designed network also meets the requirement of end-to-end BER threshold.**

Keywords*—Free Space Optics, Atmospheric Turbulence, Misalignment Fading, Topology Design, Cross-layer Optimization.

I. Introduction

Free Space Optics (FSO) refers to an optical communication technology that transmits data using a laser beam in free space between a pair of transceivers. FSO transceivers are now widely available in the market and an FSO link can be set up quickly in several minutes to hours. Contrast to the fiber-optic networks, FSO networks can be deployed without requiring to lay out physical cable. Therefore, FSO networks are a promising candidate for densely populated urban areas, where the deployment of fiber optic infrastructure is impractical due to high costs or physical deployment difficulty [1]. For example in Fig. 1, FSO transceivers could be used to setup a backbone campus network where there are a lot of tall buildings and it is not really convenient to run the cable under the ground between buildings.

Nowadays, we observe often disasters such as earthquake, tsunami, and flood which usually destroy the infrastructure of a whole region, including communication networks. FSO links could be used to setup a network in responding to the requirement of quick recovery of communication in the affected region after the disaster thanks to the good mobility of FSO transceivers and fast link deployment. FSO transceivers will be placed on some secured places and then they are aligned to see each other according to a topology for making a network. The FSO transceivers located at the same site can be wired together for exchanging the data between their FSO links.

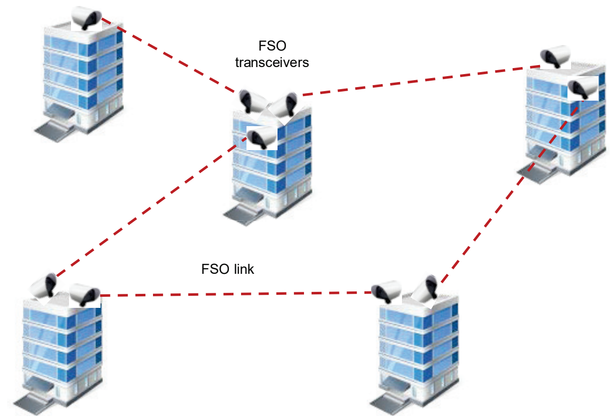

Fig. 1. An example of FSO network.

Mesh topology is a good choice for network architecture thanks to its advantages of high availability, enhanced capacity, and network utilization. The first FSO mesh network was proposed by A.S. Acampora *et al.* in [2]. In this study, the authors describe an approach that uses of FSO links to interconnect densely deployed packet-switching nodes in a multihop mesh topology. Next, a broadband access network based on FSO links in mesh architecture is proposed in [3]. Although the problem of designing wireline and RF wireless network topology has been largely studied but the problem of designing FSO mesh networks is still new. In [4], the authors consider the problem of designing a topology with strong connectivity and short diameter for FSO networks. Two centralized approaches including Delaunay triangulation and Closest Neighbor (CN) algorithms are presented in this study. As an extension of the work of [4], the authors in [5] proposed network topology design (NTD) algorithm, which is

Linh D. Truong and Toi V. Doan are with School of Information and Communication Technology, Hanoi University of Science and Technology, Hanoi, Vietnam

Hien T. T. Pham and Ngoc T. Dang are with Department of Wireless Communications, Posts and Telecommunications Institute of Technology, Hanoi, Vietnam

Ngoc T. Dang is also with Computer Communications Lab., The University of Aizu, Aizu-wakamatsu, Fukushima 965-8580, Japan

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Corresponding author: Linh. D. Truong (e-mail: linhtd@soict.hust.edu.vn).

able to achieve not only high spatial diversity but also high reliability. This work is, however, limited to the case that at least three FSO transceivers are deployed at each site. S. Chen *et al.* proposed heuristic algorithms for designing minimum cost FSO networks in [6]. The total number of links in the network or, equivalently, the number of transceiver pairs is the criterion used for minimizing cost. In [7] and [8], the design and optimization problems for the wireless access network and cellular backhaul network with FSO links are studied, respectively. The authors in [7] consider two sub-problems including the optimized clustering problem in the underlying wireless mesh network and the topology optimization problem on designing the upper tier FSO network topology. Regarding FSO-based backhaul networks, finding a cost-effective solution to upgrade the cellular backhaul with pre-deployed optical fibers using FSO links and mirror components is presented in [8]. Although design and optimization of FSO mesh network have been studied profoundly, there are still unsolved problems. The impact of impairments caused by the atmospheric channel on network design and optimization were often ignored in previous works [4],[5],[6] except [7] and [8], where the impact of weak atmospheric turbulence is quantified into the weight of an FSO link via received intensity or link reliability, respectively.

In order to build a more effective algorithm for designing and optimizing FSO networks, a cross-layer approach is developed in our study. The proposed optimization algorithm takes into account *the weight of an FSO link that is measured via bit-error rate* (BER), a practical and useful performance parameter. The link weight in our research, therefore, can reflect many factors such as the signal power, noise power, and the distance. Especially, the weight of an FSO link takes into account the effect of atmospheric turbulence. Instead of considering log-normal distributed turbulence channel as in [7], the turbulence channel in our analysis is modeled as a Gamma-Gamma distributed channel, which is widely used to consider the effect of turbulence in moderate-to-strong conditions [9],[10]. Last but not least, misalignment fading, which has strong impact on the performance of FSO link [11]-[14], is also included in the calculation of the link weight.

In building an FSO mesh network, especially restoration network for serving disaster recovery, there are two main concerns to be considered as follows: (1) network cost and (2) installation time. Since it is easy to install the FSO transceivers, the main network cost will be the cost of the transceivers. The network installation time includes the time placing FSO transceivers on site and time for aligning FSO links. In this paper, we, therefore, propose to take minimization of network cost and installation time as the optimization objectives. An optimal and a heuristic algorithms will be proposed to choose the best sites to install the FSO transceivers and also the best topology for a given traffic matrix in a region. The algorithms aim to use the least of FSO links as possible since this allows to reduce both the network costs and the installation time.

The remainder of the paper is organized as follows. Section II states the problem of designing FSO mesh networks. Section III presents how atmospheric turbulence and misalignment fading channels affect BER of FSO links. Section IV models the optimal FSO mesh network by using linear programming. Section V proposes a heuristic solution to solve the network design problem. Section VI shows the numerical results. Finally, Section VII concludes the paper.

II. Problem Formulation

In this paper, the problem of designing FSO mesh network is defined as follows.

Given

- A set of possible sites for installing FSO transceivers.
- A traffic matrix to be carried by the network. The traffic matrix is expressed in form of a list of connection demands between sites.

Assume that

- All FSO links have an identical capacity, i.e., the bit rate of all FSO links are the same.
- A link between two sites uses one FSO transceiver at each site. The capacity of transmission between the two sites in one direction is independent of the other direction.
- FSO transceivers of the same site are connected to each other by a local wired network and the impact of this network is not taken into account in this research.

We need to seek for a network topology composing of FSO transceivers and links between them so that

- *The whole demanded traffic can be carried by the network.*
- *Every FSO demanded connection must have end-to-end BER under threshold δ.*
- *The solution must minimize the cost of FSO transceivers and minimize the transceiver installation effort.*

Let the unit cost of a FSO transceiver be C_{FSO} and the number of FSO links finally included in the topology be n, then the total network cost is $2n \times C_{FSO}$.

Let us evaluate the installation effort by time and the average time to install an FSO link be T_{link}, then the total installation time of the network is $n \times T_{link}$.

It is clear that, in order to minimize the network cost and the installation time, we need only to minimize the number of FSO links in the network topology. Therefore, the optimization objective of the topology design turns to minimizing the total number of FSO links.

Let BER_ℓ be BER of link ℓ, BER_ℓ is calculated according to the channel model that will be presented in Section III. Let BER_p be the BER of an end-to-end connection p which may go through several FSO links. The probability of non-error along p is the product of the probability of non-error of all links in p and hence can be given as

$$1 - \text{BER}_p = \prod_{\ell \in p} (1 - \text{BER}_\ell). \quad (1)$$

Therefore, the constraint restricting the end-to-end BER under threshold δ becomes

$$\text{BER}_p = 1 - \prod_{\ell \in p} (1 - \text{BER}_\ell) \le \delta. \quad (2)$$

In summary, while designing the FSO network that minimizes the total number of FSO links we need to make sure that each demand of the traffic matrix takes a route that satisfies constraint (2).

III. BER Performance of an FSO Link

FSO channel considered in this study is characterized by three parameters including channel loss, atmospheric turbulence-induced fading, and pointing errors. The mathematical model of channel state can be expressed as

$$h = h_l h_a h_p, \tag{3}$$

where h_l is the channel loss coefficient. h_a represents the intensity fluctuation due to atmospheric turbulence. h_p is the fraction of power collected by a photo-detector (PD), which depends on the relative distance between the PD and the center of the received optical beam. In one bit duration, it can be assumed that h_l is deterministic while h_a and h_p are random variables.

The optical signal is attenuated while traversing atmospheric channel due to absorption and scattering processes. Signal attenuation is caused by the variation of the concentrations of matter in the atmosphere, which depend on the weather conditions. According to Beers-Lambert law, the channel loss coefficient is described as [15]

$$h_l = \exp(-a_l z), \tag{4}$$

where a_l the attenuation coefficient and z is the transmission distance.

Intensity fluctuation (or fading) happens at the receiver due to atmospheric turbulence. In this study, we use Gamma-Gamma distribution in order to investigate the system performance in moderate-to-strong turbulence regime. The probability distribution function (PDF) of the intensity fluctuation is thus given by [9]

$$f_{h_a}(h_a) = \frac{2(\alpha\beta)^{\frac{(\alpha+\beta)}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h_a^{\frac{(\alpha+\beta)}{2}-1} K_{\alpha-\beta}\left(2\sqrt{\alpha\beta h_a}\right), \tag{5}$$

where $K_v(.)$ is the modified Bessel function of the second kind and order v. $\Gamma(.)$ is the standard gamma function. The two parameters $\alpha > 0$ and $\beta > 0$ can be adjusted for wide range of turbulence conditions. In the case of planar wave propagation, they are directly linked to physical parameters as [9]

$$\alpha = \left[exp\left(\frac{0.49\sigma_R^2}{\left(1+1.11\sigma_R^{12/5}\right)^{7/6}} \right) - 1 \right]^{-1} \tag{6}$$

$$\beta = \left[exp\left(\frac{0.51\sigma_R^2}{\left(1+0.69\sigma_R^{12/5}\right)^{5/6}} \right) - 1 \right]^{-1}, \tag{7}$$

where σ_R^2 is the unitless Rytov variance, which represents the strength of the turbulence and is defined as

$$\sigma_R^2 = 1.23\left(\frac{2\pi}{\lambda}\right)^{7/6} C_n^2 z^{11/6}, \tag{8}$$

where λ is the wavelength. C_n^2 is the index of refraction structure parameter and z is the distance of an FSO link.

To compute the PDF of the fraction of power collected by a PD h_p, we use the assumptions and methodology described in [11], which assumes a circular detection aperture of radius r and a Gaussian beam. Consequently, the PDF of h_p can be derived as [11]

$$f_{h_p}(h_p) = \frac{{\gamma_p}^2}{A_0^{\gamma_p^2}} (h_p)^{\gamma_p^2-1}, \tag{9}$$

where $\gamma_p = \omega_{z_{eq}}/2\sigma_s$ is the ratio between the equipment beam radius and the jitter standard deviation σ_s of the misalignment. The parameter $\omega_{z_{eq}}$ can be calculated using the relations $v = \sqrt{\pi}r/\sqrt{2}\omega_z$, $A_0 = [erf(v)]^2$ and $\omega_{z_{eq}}^2 = \omega_z^2\sqrt{\pi}\text{erf}(v)/2v\exp(-v^2)$, where erf(.) is the error function and ω_z is the beam waist (radius calculated at e^{-2}) at the distance z.

By expressing the $K_v(.)$ in terms of Meijers G-function and making simplification, the PDF of the channel state, $h = h_l h_a h_p$, is given as [16]

$$f_h(h) = \frac{\alpha\beta{\gamma_p}^2}{A_0 h_l \Gamma(\alpha)\Gamma(\beta)} G_{1,3}^{3,0}\left(\frac{\alpha\beta}{A_0 h_l} h \middle| \begin{matrix} {\gamma_p}^2 \\ {\gamma_p}^2-1, \alpha-1, \beta-1 \end{matrix}\right). \tag{10}$$

The signal-to-nosie ratio (SNR) of the links between two FSO transceivers is defined as follows

$$\gamma = \frac{{P_T}^2 \Re^2 h^2}{\sigma_n^2}, \tag{11}$$

where γ denotes the instantaneous SNR of the link between transmitter and receiver. P_T is the transmitted optical power, σ_n^2 is variance of additive white Gaussian noise (AWGN). $\Re$ is the responsivity of the photodetector.

Denoting $p(1)$ and $p(0)$ are the probabilities of sending bit "1" and bit "0", the instantaneous BER of an FSO link using OOK is given by $P_{e,FSO}(h) = p(1)\,p(e|1,h) + p(0)\,p(e|0,h)$, where $p(e|1,h)$ and $p(e|0,h)$ are the conditional bit error probabilities corresponding to the cases that bit "1" and bit "0" are transmitted, respectively. Assuming that $p(1) = p(0) = 1/2$ and $p(e|1,h) = p(e|0,h)$, the instantaneous BER of FSO link can be computed as

$$P_{e,FSO}(h) = p(e|1,h) = p(e|0,h) = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{\gamma}{2}}\right), \tag{12}$$

where erfc(.) is the complementary error function, whose closed-form expression can be expressed via Meijer's G–function as $\text{erfc}(x) = \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left(x \middle| \begin{matrix} 1 \\ 0, 1/2 \end{matrix}\right)$ [[17],Eq. (06.27.26.0006.01)]. γ is the SNR of FSO link, which is derived from (11). The average BER can be obtained by averaging (12) over the PDF of h as follows

$$\text{BER} = \int_0^\infty f_h(h) P_{e,FSO}(h)\,dh. \tag{13}$$

By substituting (10), (11), and (12) in (13) and expressing efrc(.) as Meijer's G–function, the extract-form expression for the average BER of FSO link is given by (14). Next, using [[18], Eq. (21)] and [[19], Eq. (9.31.1)] the closed-form expression for the average BER can be expressed as (15).

$$\text{BER} = \frac{\alpha\beta\gamma_p^2}{A_0 h_l \Gamma(\alpha)\Gamma(\beta)} \int_0^\infty G_{1,3}^{3,0}\left(\frac{\alpha\beta}{A_0 h_l} h \middle| \begin{matrix} \gamma_p^2 \\ \gamma_p^2-1, \alpha-1, \beta-1 \end{matrix}\right) \frac{1}{2\sqrt{\pi}} G_{1,2}^{2,0}\left(\frac{\Re^2 P_t^2}{\sigma_1^2} h^2 \middle| \begin{matrix} 1 \\ 0, 1/2 \end{matrix}\right) dh. \quad (14)$$

$$\text{BER} = \frac{2^{\alpha+\beta-3}\gamma_p^2\alpha\beta}{\sqrt{\pi^3} A_0 h_l \Gamma(\alpha)\Gamma(\beta)} G_{6,3}^{2,5}\left(\frac{16\Re^2 P_t^2 A_0^2 h_l^2}{\sigma_1^2 \alpha^2 \beta^2} \middle| \begin{matrix} -\frac{\gamma_p^2-2}{2}, \frac{-\alpha+1}{2}, \frac{-\alpha+2}{2}, \frac{-\beta+1}{2}, \frac{-\beta+2}{2}, 1 \\ 0, 1/2, -\gamma_p^2/2 \end{matrix}\right). \quad (15)$$

IV. Optimization Solution with ILP Model

Given the complexity of the design problem, in order to seek for the optimal FSO mesh network, we define an Integer Linear Programming (ILP) that models the optimal network. We will later use a ILP solver tool for solving this ILP model. Following notations are used in the model.

A. Notations

- $\mathbb{M}$ is the set of possible sites for installing FSO transceivers. Assume that it is possible to install as many FSO transceivers as needed at each site.
- An FSO link between site i and site j is represented by a pair of directional links from i to j, denoted by (i, j), and from j to i, denoted by (j, i).
- $\mathbb{D} = \{(s, t, d_{st}) : s, t \in \mathbb{M}\}$ is the set of connection demands between sites. Each demand in $\mathbb{D}$ is represented by tuple (s, t, d_{st}), where $s \in \mathbb{M}$ and $t \in \mathbb{M}$ are the source and the destination of the demand and d_{st} is the demanded bandwidth. The demand is assumed perfectly divisible so it can be carried over multiple flows between s and t.
- BER_{ij} is the BER of the link between two FSO transceivers if they are placed at site i and site j. It can also be denoted as BER_ℓ, where ℓ is the link. BER is calculated according to (15).
- $f(\alpha)_{ij}^{st}$ is the portion of bandwidth of the demand (s, t, d_{st}) that is carried by flow indexed α of over link (i, j).
- $x(\alpha)_{ij}^{st}$ is a binary variable, it takes value 1 if flow indexed α of demand (s, t, d_{st}) uses link (i, j) and 0 otherwise.
- x_{ij} is a binary variable, $x_{ij} = 1$ if the physical FSO link between i and j should be included in the topology, and 0 otherwise.
- r is capacity of a FSO link

According to the problem statement in Section II, the objective of the ILP model is to look for a set of links so that:

- All traffic demands in $\mathbb{D}$ are accommodated using some flows and the end-to-end BER of each flow is under threshold δ
- The number of links in the topology is minimized.

B. ILP Model

1) Flow conservation constraints for each demand: Since each demand $(s, t, d_{st}) \in \mathbb{D}$ could be routed over different flows (indexed by α) the following constraints assure that each flow forms a path from s to t and the total bandwidth carried by all flows is exactly d_{st}:

$$\sum_{\forall\alpha}\sum_{\forall j\in\mathbb{M}} (f(\alpha)_{sj}^{st} - f(\alpha)_{js}^{st}) = d_{st} \quad (16)$$

$$\sum_{\forall\alpha}\sum_{\forall i\in\mathbb{M}} (f(\alpha)_{it}^{st} - f(\alpha)_{ti}^{st}) = d_{st} \quad (17)$$

$$\sum_{\forall i\in\mathbb{M}} f(\alpha)_{ik}^{st} - \sum_{\forall j\in\mathbb{M}} f(\alpha)_{kj}^{st} = 0, \forall k \neq s, t, \forall\alpha \quad (18)$$

2) Capacity constraint on each link:

$$\sum_{\forall\alpha}\sum_{\forall d_{st}\in\mathbb{D}} f(\alpha)_{ij}^{st} \leq r, \forall i, j \in \mathbb{M} \quad (19)$$

3) End-to-end BER constraint: Constraint (2) is equivalent to

$$\log(\prod_\ell (1 - \text{BER}_\ell)) \geq \log(1-\delta)$$

thus

$$\sum_\ell \log(1 - \text{BER}_\ell) \geq \log(1-\delta)$$

The following constraints enforce the end-to-end BER constraint to every flow:

$$x(\alpha)_{ij}^{st} \geq \frac{f(\alpha)_{ij}^{st}}{d_{st}} \quad \forall (s, t, d_{st}) \in \mathbb{D}, \forall\alpha \quad (20)$$

$$\sum_{ij} \log(1 - \text{BER}_{ij}) x(\alpha)_{ij}^{st} \geq \log(1-\delta), \quad \forall (s, t, d_{st}) \in \mathbb{D}, \forall\alpha \quad (21)$$

4) Auxiliary constraints: Physical FSO link between site i and site j should be included in the topology if there is at least one flow going from i to j or from j to i

$$x_{ij} \geq \frac{1}{\sum_{st} d_{st}} \sum_{st}\sum_{\alpha} (x(\alpha)_{ij}^{st} + x(\alpha)_{ji}^{st}), \quad \forall i, j \in M, d_{st} \in \mathbb{D} \quad (22)$$

5) Objective function:

$$\min \sum_{i,j\in M} x_{ij} \quad (23)$$

In all simulations of this paper, we set parameter $\alpha = 1$ so that each demand is routed over a single path.

V. Heuristic Solution

The main idea of the heuristic algorithm is that the network is built gradually while seeking a connection path for each demand in the given traffic matrix. The list of demands in $\mathbb{D}$ will be browsed one after the other. For each demand, we will seek a path that has sufficient bandwidth for the demand and goes through the least of FSO links. For finding the path, we use Dijkstra algorithm. In order to make sure that the end-to-end BER of each path does not exceed threshold δ, the Dijkstra algorithm is modified so that it checks the end-to-end BER constraint at each iteration. The modified Dijkstra algorithm is presented in Algo. 1. The FSO links of the selected path are then included in the topology. The subsequent demands are routed similarly but the links already included in the topology will be prioritized to be used. In so doing, we can approach the objective of minimizing the total number of FSO links.

Details of the heuristic algorithm are as follows. Let $\mathbb{T}$ be the topology to be built. Let $\mathbb{G}$ be directed full graph made from all FSO sites in $\mathbb{M}$. There are two arcs in opposite directions between each pair of FSO sites representing the capability to transport data from one site to the other. The maximum capacity of each arc is r. All arcs with BER greater than threshold δ are removed from the graph. The algorithm follows below main steps:

Initiation:

- $\mathbb{T} = \emptyset$,
- Assign weight 1 to all arcs of $\mathbb{G}$.

Loop for each demand $(s, t, d_{st}) \in \mathbb{D}$:

1) Exclude from $\mathbb{G}$ arcs with insufficient bandwidth for the demand d_{st}.
2) Route the demand over the shortest path in $\mathbb{G}$ by using the Shortest path algorithm with constraint in Algo 1. In so doing, the demand will take the route with the least of FSO links thus using the least of FSO transceivers while the end-to-end BER constraint (2) is guaranteed.
3) Subtract d_{st} from the residual bandwidth of all arcs in the path. Include the links along the path to the topology by adding the arcs of the path and their reversed arcs to $\mathbb{T}$.
4) Update weights for all arcs $(i, j) \in \mathbb{T}$ by $1 - \frac{bw_{ij}}{c_{ij}}$, where bw_{ij} is the residual bandwidth of arc (i, j). This weight does not exceed 1 thus all arcs of $\mathbb{T}$ will be prioritized in the next path-finding. The priority level degrades when the residual bandwidth of arc reduces.
5) Repeat the process until all demands are routed.

In the modified Dijkstra algorithm in Algo 1, at each vertex browsing step, function `BER-e2e`(path) checks if the end-to-end BER of the current path is under threshold δ according to constraint (2).

In order to improve the efficiency of the algorithm, in the initial step, we sort all demands in $\mathbb{D}$ in two groups: i) the group of demands whose source and destination are close enough to be directly connected via a single FSO link (under the transceiver operation range) and ii) the group of remaining demands. The demands in the first group are routed before the second group. In each group, demands are sorted again in decreasing order of requested bandwidth.

Algorithm 1: Shortest path with constraint

```
Data: 𝔾, s
Result: list of distance, list of path
Q ← vertex set of 𝔾;
foreach vertex v ∈ Q do
    dist[v] ← INFINITY;
    prev[v] ← UNDEFINED;
dist[s] ← 0;
while Q ≠ ∅ do
    u ← vertex in Q with min dist[u];
    foreach neighbor v of u do
        alt ← dist[u] + length(u, v);
        if alt < dist[v] and BER-e2e (prev[v]) then
            dist[v] ← alt;
            prev[v] ← u;
return dist[], prev[] ;
Function BER-e2e(path):
    prod ← 1;
    for ℓ ∈ path do
        prod ← BERe2e ×(1 − BER_ℓ);
    if prod > 1 − δ then
        return true;
    else
        return false;
```

VI. Numerical Results

The proposed heuristic algorithm has been implemented in Matlab. The optimal ILP model described in Section IV has also been solved by using ILOG Cplex.

Practically, FSO communication systems often employ forward error correction (FEC) to improve the BER performance. An FEC is able to correct the maximum BER of 10^{-3} to better than 10^{-12}, which can be considered as error-free communications. In our analysis, as the BER performance is evaluated for the case of without FEC, and the end-to-end BER threshold is, therefore, set to the maximum value of 10^{-3}. System parameters used in our analysis are shown in Table I.

FSO sites are generated randomly in the experimental space of $S \times S \times 100$ m, where S varies in range $2000 \sim 6000$ m. The variance in height is only 100 m in order to simulate the height of the high building where FSO transceivers could be. The traffic demands in $\mathbb{D}$ are generated with sources and destinations taken randomly from $\mathbb{M}$ sites and the demanded bandwidth d_{st} varies between $100 \sim 300$ Mbps.

The performance of the proposed algorithms are evaluated regarding the total number of links they use for building the FSO networks since this parameter reflects the network costs. The evaluation are performed with different network sizes, network loads and site densities. For defining network load, we introduce the notion of feasible link.

Feasible links are links between sites with a distance under the operation range L_{max} of FSO transceivers. We denote the number of feasible links of a network by $n_{feasible}$

TABLE I. SYSTEM PARAMETERS

Name	Symbol	Value
Link's bit rate	R_b	1 Gbps
Wavelength	λ	1550 nm
Refractive index structure coeff.	C_n^2	10^{-14} $\mathrm{m}^{-2/3}$
Transmitted power	P_T	0 dBm
Noise variance	σ_n^2	10^{-19} A^2
Attenuation coefficient	a_l	0.1 km^{-1}
Receiver diameter	$2a$	20 cm
Beam radius at 1 km	ω_z	2.0 m
Beam divergence angle	θ	1 mrad
Jitter standard deviation	σ_s	10 cm
End-to-end BER threshold	δ	10^{-3}
Number of FSO sites	$\mathbb{M}$	$5 \sim 90$
Demanded bandwidth per connection	d_{st}	$100 \sim 300$ Mbps
Experimental space $S \times S \times 100$ m	S	$2000 \sim 5000$ m
Density of FSO sites	ρ	$1 \sim 3$ sites /km^2
Operation range of the FSO transceiver	L_{max}	1600 m

Network load is the ratio of occupied bandwidth on all network links over the total capacity of all feasible links. Network load varies in range $[0..1]$.

Given a demanded traffic matrix between network nodes, network load incurred by this traffic matrix can only be exactly calculated once the routes of all traffic demands are identified. Without the routing, we can only estimate the network load. Below is an estimation of network load of a given traffic matrix by the lower bound of the load that may incurred.

Let λ_{ij} be the total requested bandwidth from FSO site i to FSO site j. Let l_{ij} be the Euclid distance between sites i and j. The traffic between sites i and j should take at least $\lceil \frac{l_{ij}}{L_{max}} \rceil$ links, thus it will use in total at least the following amount of bandwidth along its path: $\lambda_{ij} \times \lceil \frac{l_{ij}}{L_{max}} \rceil$

Since the total capacity of network links is $2.r.n_{feasible}$, the lower bound of network load incurred by traffic matrix $\mathbb{D}$, denoted by $u(\mathbb{D})$, is

$$u(\mathbb{D}) = \frac{\sum_{(i,j)\in\mathbb{D}} \lambda_{ij} \times \lceil \frac{l_{ij}}{L_{max}} \rceil}{2.r.n_{feasible}} \tag{24}$$

This value $u(\mathbb{D})$ gives us an idea of the minimum load of the network when accommodating all traffic matrix $\mathbb{D}$.

A. Comparison between the Heuristic and Optimal solutions in small size datasets

This comparison will provide an idea of how algorithms choose links for including in the topology and also how good the heuristic algorithm is in designing the topology. In principle, the ILP model needs to browse all possible topologies to find the optimal one thus it takes a long time to run on a large set of FSO sites and even does not give results when the number of sites in $\mathbb{M}$ becomes too important. Therefore, we compare the proposed heuristic algorithm with the optimal algorithm by ILP only in small network instances with few number of FSO sites.

Figures 2 and 3 show examples of the topology designed by the proposed heuristic and the optimal topology designed by ILP model for a set of $\mathbb{M} = 10$ sites distributed within an area of 3000m × 3000m × 100m. The traffic matrix to be carried by the network contains $\mathbb{D} = 45$ requests between sites. The topology designed by the heuristic algorithm uses 15 links while the optimal topology uses 11 links.

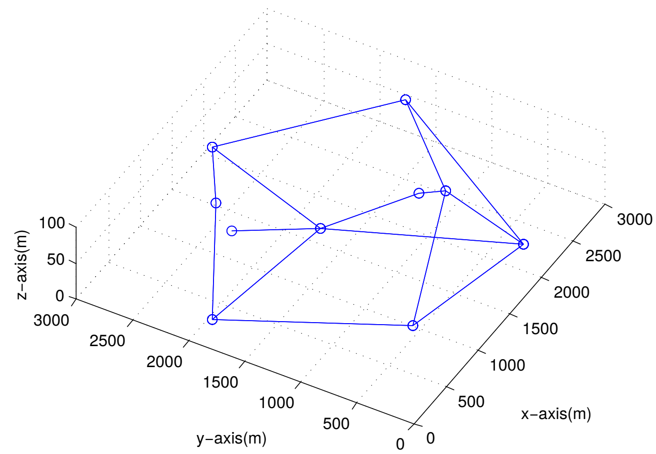

Fig. 2. 3D view of the topology generated by the heuristic algorithm with $S = 3000, \mathbb{M} = 10, \mathbb{D} = 45$. The topology contains 15 links.

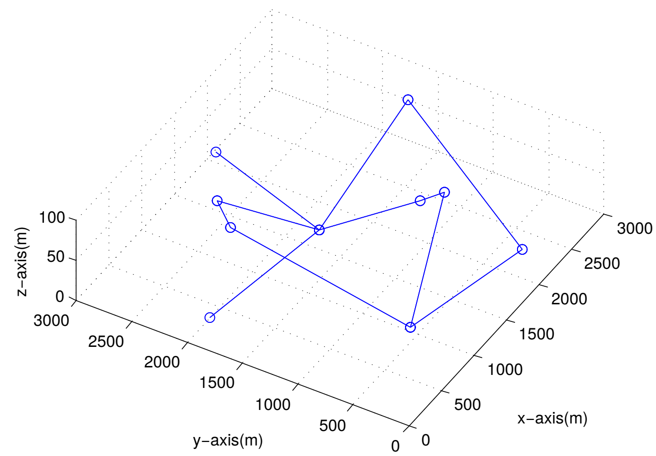

Fig. 3. 3D view of the optimal topology generated by the ILP algorithm with $S = 3000, \mathbb{M} = 10, \mathbb{D} = 45$. The topology contains 11 links.

Figure 4 shows the calculation time of the proposed heuristic and ILP model when the number of FSO sites increases from 5 to 20 sites. The execution time of the ILP model grows up quickly in comparison with the execution time of the heuristic. When the number of FSO sites $\mathbb{M} > 20$, we even cannot get results from ILP model while the heuristic takes few seconds for even 30 sites.

Figure 5 shows the numbers of links of the topologies designed by the heuristic algorithm and the ILP model when the number of sites $\mathbb{M}$ increases while site density varies between $1 \sim 1.5$. The figure shows also the number of feasible links in the datasets. The ILP model provides the optimal FSO networks with the fewest links. With a small number of sites $\mathbb{M} = 5$ and $\mathbb{M} = 7$, the heuristic finds also the optimal topologies. In the remaining cases, the gap between the optimal

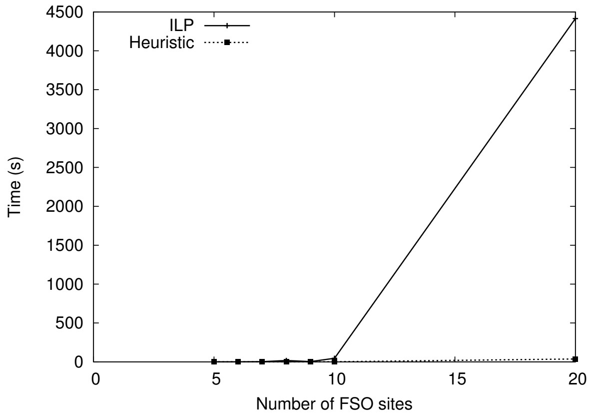

Fig. 4. Calculation time of the proposed heuristic algorithm and ILP model.

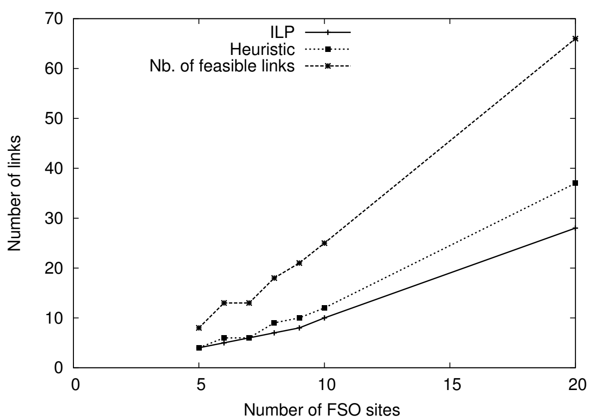

Fig. 5. The numbers of links in the topologies designed by the proposed heuristic algorithm and ILP model when the number of FSO sites increases.

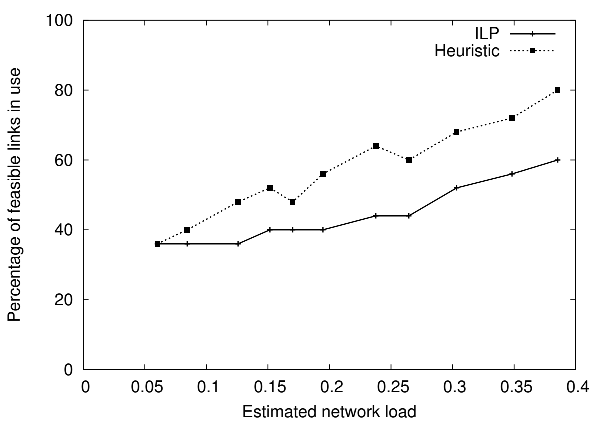

Fig. 6. Percentage of feasible links included in the topology with a fixed set of $\mathbb{M} = 10$ sites when the network load increases.

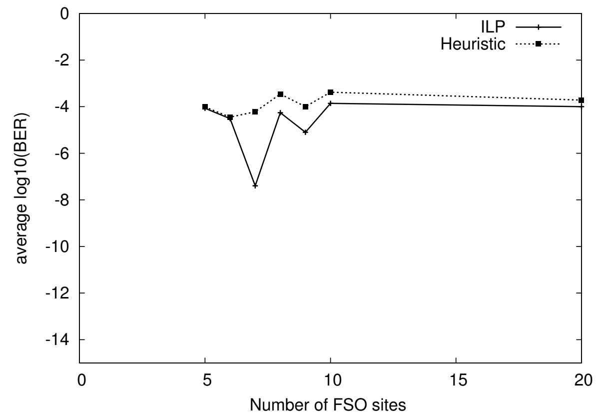

Fig. 7. Average value of $\log_{10}$ of BER of links in the topology designed by the proposed heuristic algorithm and ILP model.

topology and the heuristic designed topology increases with the number of FSO sites.

Figure 6 shows the percentage of feasible links used in the topology when the set of possible sites is fixed with $\mathbb{M} = 10$ and network load increases from 0.05 to 0.4 by adding gradually requests in the traffic matrix $\mathbb{D}$. The *percentage of feasible links used* is defined as the ratio between the number of links included in the designed topology and the total number of feasible links. The proposed heuristic uses at most 20% of feasible links more than the optimal solution. This number shows the efficiency of the heuristic algorithm in small size networks. With network load higher than 0.4 the problem becomes infeasible according to the report of ILP model. It worth to note that even though the maximum network load is 1, this load can hardly be achieved. The reason is that, in low-density FSO sites, there are few feasible links in the network consequently some links are critical for several traffic paths. These links may become full quickly thus cannot serve for all required traffic.

Figure 7 shows the average value BER of links in the topology. In the worst cases, average BER of links is around 10^{-4}. This value is quite smaller than the BER tolerability of FSO transceiver, which is targeted at BER of 10^{-3} so that, the error-free communications can be guaranteed with the use of FEC.

B. Evaluation of the Heuristic under different network loads

In this session, we analyze the behavior of the heuristic algorithm under different network sizes and network loads. For each network size, we first generate a set of FSO transceivers that are distributed with density of FSO transceivers varying in the range of $\rho = 1.2 - 1.7$ sites per projected square kilometer (site elevation is ignored).

- $\mathbb{M} = 15, S = 3000$ m, $\rho = 1.67$
- $\mathbb{M} = 20, S = 4000$ m, $\rho = 1.25$
- $\mathbb{M} = 30, S = 5000$ m, $\rho = 1.2$
- $\mathbb{M} = 40, S = 5000$ m, $\rho = 1.6$

TABLE II. ESTIMATED NUMBER OF LINKS USED BY A CONNECTION

S (m)	$\mathbb{M}$	ρ	estimated nb. links/conn.	actual nb. links/conn.
3000	15	1.67	1.007	2.133
4000	20	1.25	1.329	2.559
5000	30	1.2	1.572	3.207
5000	40	1.6	1.572	3.077

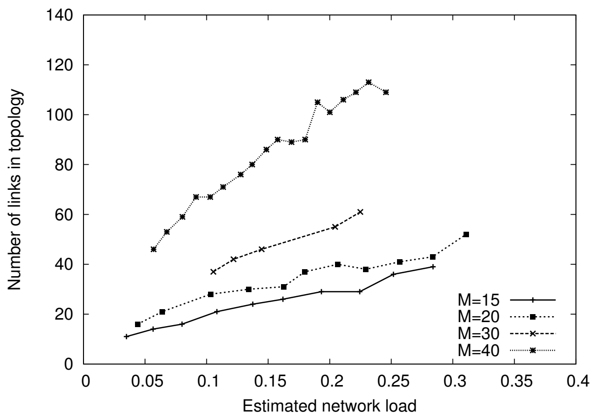

Fig. 8. Number of links used in the designed topology versus the network load.

For each network size, we generate traffic matrices $\mathbb{D}$ for that network so that the network load grows gradually from 0.03 to 0.4.

The curves in Figure 8 shows the number of links involved in the topology designed by heuristic in each case for different network load. Obviously, when the network load increases, more links need to be involved. On the other hand, with the same network load level, the more FSO sites the more links are involved.

Looking deeper into how many of links would be involved in a topology, we produced Figure 9 that shows the dependancy between percentage of feasible links used in the topology and network load. We can see that *the percentage of feasible links required in a topology seems not depend on the number of available FSO sites but depends only on network load*. In this simulation, the relationship between the network load and the percentage of used feasible links is roughly described by line $f(x) = 200x + 15$. That means 10% of more network load required around 20% more of links. Hence, 100% of feasible links would be required when the estimated network load approaches 0.425. In other words, network load cannot go higher than 42%. The reason is that the estimated network load calculated in (24) is based on the assumption that all FSO links have maximum length L_{max}, therefore a connection between sites i and j is estimated to take only $\lceil \frac{l_{ij}}{L_{max}} \rceil$ FSO links. However, a connection actually goes through more number of links. The statistics in Table II shows that in average a connection takes 2 times more links than estimation. Besides, Figure 10 also shows that the average number of links used by connection is roughly constant with the variation of network loads.

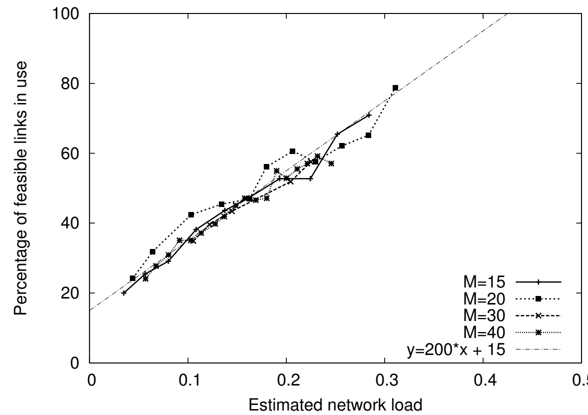

Fig. 9. Percentage of feasible links used in the topology versus the network load.

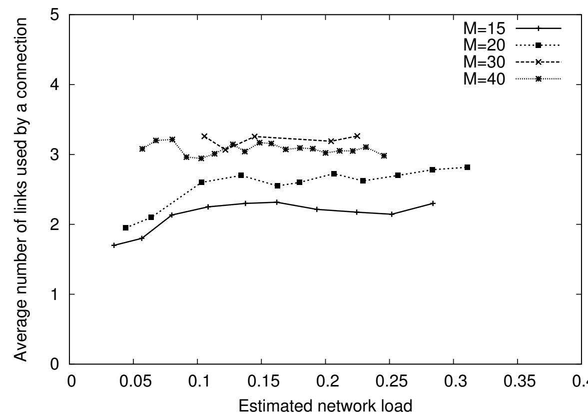

Fig. 10. Number of links per connections versus the network load.

C. Evaluation of the Heuristic with different FSO densities

The simulation in this session aims to identify the minimum density of FSO sites for being able to accommodate all requested traffic between sites. For this purpose, we start with a fixed region of $S \times S \times 100$ m, a fixed traffic matrix between all network nodes and a small number of FSO sites. Then FSO sites are added gradually in the region for increasing the density of sites. The network load thus decreases since the number of feasible links increases. Questions are: i) from which density all requested traffic could be accommodated ii) what is the maximum network load that a network can accept.

The simulation is performed with 3 network sizes with the initial number of sites make minimum density $\rho = 1$:

- S = 4000 m, initially $\mathbb{M} = 15, \mathbb{D} = 100$ requests, equivalent to initial $\rho = 0.94$ and initial network load $u(\mathbb{D}) = 0.4$
- S = 5000 m, initially $\mathbb{M} = 25, \mathbb{D} = 130$ requests, equivalent to initial $\rho = 1$ and initial network load $u(\mathbb{D}) = 0.34$
- S = 6000 m, initially $\mathbb{M} = 36, \mathbb{D} = 200$ requests,

TABLE III. LOWEST FEASIBLE FSO DENSITY AND HIGHEST FEASIBLE NETWORK LOAD FOR DIFFERENT NETWORK SIZES

S	initial number of sites $\mathbb{M}$	number of demands $\mathbb{D}$	lowest feasible density ρ	highest feasible network load $u(\mathbb{D})$
4000	15	100	1.56	0.16
5000	25	130	1.32	0.20
6000	36	200	1.72	0.12

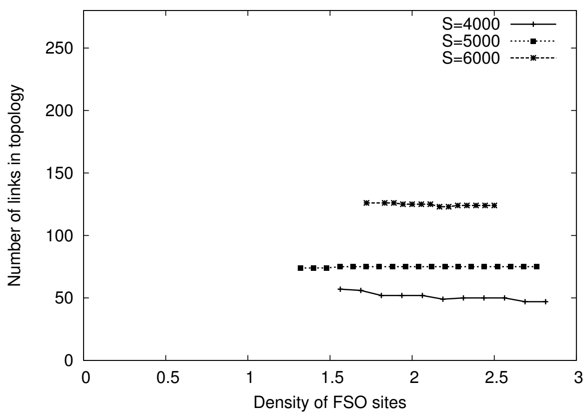

Fig. 11. Total number of links in the designed topology versus the density of FSO sites with a constant traffic matrix $\mathbb{D}$.

equivalent to initial $\rho = 1$ and initial network load $u(\mathbb{D}) = 0.32$

In these tests, we remark also that the actual length (in terms of the number of links) of a connection is 1.8-2.5 times longer than the estimated length in (24), that means an estimated network load of 0.4 will occupy actually 70%–100% network capacity. These numbers explain why the problem becomes infeasible at the load of 0.3 and up.

Figure 11 shows the total number of links in the designed topology versus the density of FSO sites under constant traffic matrix. Although the simulation started with density $\rho = 1$, the problem becomes solvable when ρ reaches $1.3 \sim 1.8$. We can see also that the number of links of the designed topology is nearly constant or slightly reduced with the increment of FSO sites. In other words, given a fix traffic matrix, we need only to put enough FSO sites for carrying the traffic matrix. Adding more FSO sites does not necessarily help to get a better topology but simply costs more in terms of FSO transceiver and installation effort. Figure 12 also confirms this affirmation as the ratio between the number of links used by the topology and the number of feasible links reduces with the increment of FSO site density. We believe that with the transceiver operation range of 1600 m, the best density would be 1.8 transceivers per projected kilometer square.

Figure 13 shows that FSO sites density has a minor impact on the number of links that a connection goes through. Adding more FSO site allows choosing a more direct path between a pair of source and destination so reduce the number of links to use. However, this impact is negligible.

Finally, according to Table III, the maximum network load could be attained is roughly 0.2, at the lowest FSO density where the algorithm accommodates successfully all demands in traffic matrix. This means that only a number of feasible links are be included in the topology even in the lowest FSO density. Figure 12 illustrates this phenomenon as, at the lowest density (around point 1.5), only about 50–60% of feasible links are included in the topology in all test cases.

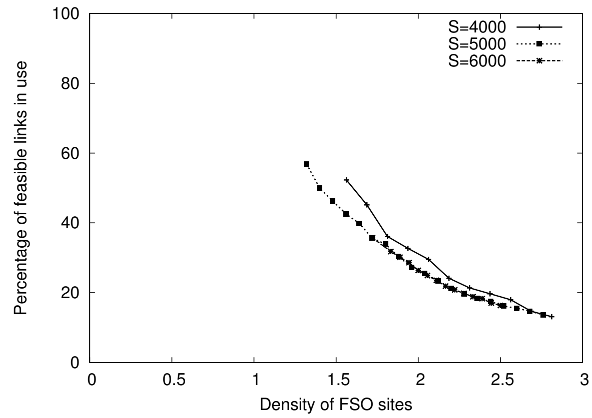

Fig. 12. Percentage of feasible links used in the topology versus the density of FSO sites with a constant traffic matrix $\mathbb{D}$.

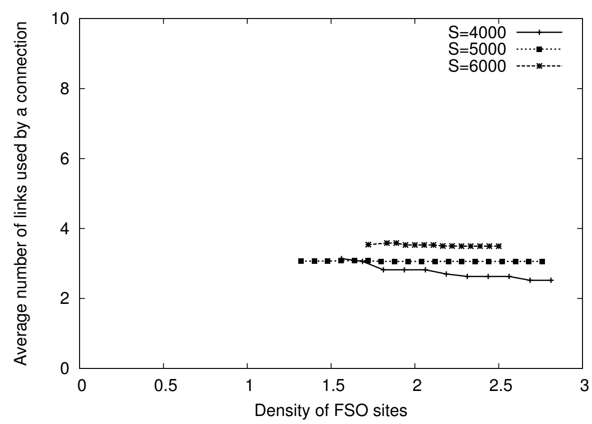

Fig. 13. Average number of links used by a connection versus the density of FSO sites with a constant traffic matrix $\mathbb{D}$.

VII. CONCLUSIONS

In this paper, we have studied the problem of designing an FSO mesh network under the effects of atmospheric turbulence, misalignment fading, and noise. A heuristic algorithm was proposed to choose the best sites to install the FSO transceivers and also the best topology for a given traffic matrix in a region. The objective of the algorithm is to minimize the network costs by using the least of FSO links as possible. In addition, the algorithm takes into account the influence of physical layer impairments ensuring that the end-to-end BER of a connection falls under the threshold of error-free communications. The simulation results demonstrated the advantage of the proposed heuristic algorithm compared to the optimal solution by ILP model in terms of short calculation time meanwhile the topology designed by the proposed heuristic uses in maximum 20% more links than the optimal solution. The simulation with the heuristic algorithm shows also that at least 1.8 FSO sites should be deployed on each square kilometer in order to be able to carry required traffic between sites. With the ability to run very fast, even with a large number of choices of FSO sites, it is practical to use the proposed heuristic algorithm in building quickly a restoration communication networks at the event of disaster.

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REFERENCES

[1] C. C. Davis, I. I. Smolyaninov, and S. D. Milner, "Flexible optical wireless links and networks," *IEEE Communications Magazine*, vol. 41, no. 3, pp. 51–57, Mar. 2003.

[2] A. S. Acampora and S. V. Krishnamurthy, "A broadband wireless access network based on mesh-connected free-space optical links," *IEEE Personal Communications*, vol. 6, no. 5, pp. 62–65, Oct. 1999.

[3] J. Zhang, "Proposal of free space optical mesh network architecture for broadband access," in *Proc. 2002 IEEE International Conference on Communications* (ICC 2002), vol. 4, 2002, pp. 2142–2145.

[4] P. C. Gurumohan and J. Hui, "Topology design for free space optical networks," in *Proc. 2003 the 12th International Conference on Computer Communications and Networks* (ICCCN 2003), Oct 2003, pp. 576–579.

[5] Z. Hu, P. Verma and J. James Sluss, "Improved reliability of free-space optical mesh networks through topology design," OSA J. Opt. Netw., vol. 7, no. 5, pp. 436–448, May 2008.

[6] S. Chen, S. Cheng, P. Verma, and R. Huck, "Heuristic algorithms for designing minimum cost fso networks," in *Proc. 2009 IEEE 3rd International Symposium on Advanced Networks and Telecommunication Systems* (ANTS 2009), Dec 2009, pp. 1–3.

[7] I. Son, S. Mao and S. K. Das, "On the design and optimization of a free space optical access network," *Optical Switching and Networking*, vol. 11, Part A, pp. 29–43, 2014.

[8] Y. Li, N. Pappas, V. Angelakis, M. Pioro, and D. Yuan, "Optimization of free space optical wireless network for cellular backhauling," *IEEE Journal on Selected Areas in Communications*, vol. 33, no. 9, pp. 1841–1854, Sept. 2015.

[9] M. A. Al-Habash, L. C. Andrews, and R. L. Phillips, "Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media," *Optical Engineering*, vol. 40, no. 8, pp. 1554–1562, 2001.

[10] Tian Cao, Ping Wang, Lixin Guo, Bensheng Yang, Jing Li, and Yintang Yang, "Average bit error rate of multi-hop parallel decode-and-forward-based FSO cooperative system with the max-min criterion under the gamma-gamma distribution," *Chin. Opt. Lett.*, vol. 13, iss. 8, pp.080101(5), 2015.

[11] A. A. Farid and S. Hranilovic, "Outage capacity optimization for free-space optical links with pointing errors," *IEEE/OSA J. Lightw. Technol.*, vol. 25, no. 7, pp. 1702–1710, Jul 2007.

[12] X. Song, F. Yang and J. Cheng, "Subcarrier intensity modulated optical wireless communications in atmospheric turbulence with pointing errors," *IEEE/OSA Journal of Optical Communications and Networking*, vol. 5, no. 4, pp. 349–358, April 2013

[13] R. Boluda-Ruiz, A. Garcia-Zambrana, C. Castillo-Vazquez, and B. Castillo-Vazquez, "Adaptive selective relaying in cooperative free-space optical systems over atmospheric turbulence and misalignment fading channels," *OSA Opt. Express*, vol. 22, iss. 13, pp. 16629–16644, 2014.

[14] P. Wang et al., "Performance Analysis for Relay-Aided Multihop BPPM FSO Communication System Over Exponentiated Weibull Fading Channels With Pointing Error Impairments," *IEEE Photonics Journal*, vol. 7, no. 4, pp. 1–20, Aug. 2015.

[15] S. Karp, R. M. Gagliardi, S. E. Moran, and L. B. Stotts, *Optical Channels: Fibers, Clouds, Water and the Atmosphere*. New York, 1998.

[16] H. T. T. Pham, N. T. Dang, and A. T. Pham, "Effects of atmospheric turbulence and misalignment fading on performance of serial-relaying M-ary pulse-position modulation free-space optical systems with partially coherent gaussian beam," *IET Communications*, vol. 8, no. 10, pp. 1762–1768, July 2014.

[17] Wolfram, "The wolfram functions site," [Online] Available at: http://functions.wolfram.com.

[18] V. S. Adamchik and O. I. Marichev, "The algorithm for calculating integrals of hypergeometric type functions and its realization in reduce system," in *Proc. the International Symposium on Symbolic and Algebraic Computation*, ser. ISSAC 90. New York, NY, USA: ACM, 1990, pp. 212–224.

[19] I. R. I.S. Gradshteyn, *Table of integrals, series, and products,* 7th ed. Academic, New York, 2007.