Artificial Intelligence

For HEDSPI Project

Lecturer 10 – First Order Logic

Lecturers :
Le Thanh Huong
Tran Duc Khanh
Dept of Information Systems
School of Information and Communication Technology - HUST

First Order Logic

- Syntax
- Semantic
- Inference
  - Resolution

First Order Logic (FOL)

- First Order Logic is about
  - Objects (e.g., people, houses)
  - Relations(e.g., red, bigger than, father of)
  - Facts
- The world is made of objects
  - *Objects* are things with individual identities and properties to distinguish them
  - Various *relations* hold among objects. Some of these relations are functional
  - Every fact involving objects and their relations are either *true* or *false*

FOL

- Syntax
- Semantic
- Inference
  - Resolution

FOL Syntax

- Symbols
  - Variables: x, y, z,…
  - Constants: a, b, c, …
  - Function symbols (with arities): f, g, h, …
  - Relation symbols (with arities): p, r, s
  - Logical connectives: $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$
  - Quantifiers: $\exists, \forall$

5

FOL Syntax

- Variables, constants and function symbols are used to build terms
  - X, Bill, FatherOf(X), …
- Relations and terms are used to build predicates
  - Tall(FatherOf(Bill)), Odd(X), Married(Tom,Marry), Loves(Y,MotherOf(Y)), …
- Predicates and logical connective are used to build sentences
  - Even(4)
  - $\forall$X. Even(X) $\Rightarrow$ Odd(X+1)
  - $\exists$X. X > 0

6

FOL

- Syntax
- Semantic
- Inference
  - Resolution

7

FOL Semantic

- Variables
  - Objects
- Constants
  - Entities
- Function symbol
  - Function from objects to objects
- Relation symbol
  - Relation between objects
- Quantifiers
  - $\exists x.P$ true if P is true under some value of x
  - $\forall x.P$ true if P is true under every value of x
- Logical connectives
  - Similar to Propositional Logic

8

Example

- Symbols
  - Variables: x,y,z, …
  - Constants: 0,1,2, …
  - Function symbols: +,*
  - Relation symbols: >, =
- Semantic
  - Universe: N (natural numbers)
  - The meaning of symbols
    - Constants: the meaning of 0 is *the number zero, …*
    - Function symbols: the meaning of + is *the natural number addition*, …
    - Relation symbols: the meaning of > is *the relation greater than*, …

Transform sentences to FOL

a. John owns a dog
∃ x. D(x) ∧ O(J,x)
D(Fido) ∧ O(J, Fido)

b. Anyone who owns a dog is a lover-of-animals
∀ x. (∃ y. D(y) ∧ O(x,y)) → L(x)
∀ x. (¬∃ y. (D(y) ∧ O(x,y)) ∨ L(x)
∀ x. ∀ y. ¬(D(y) ∧ O(x,y)) ∨ L(x)
∀ x. ∀ y. ¬ D(y) ∨ ¬ O(x,y) ∨ L(x)
¬ D(y) ∨ ¬ O(x,y) ∨ L(x)

c. Lovers-of-animals do not kill animals
∀ x. L(x) → (∀ y. A(y) → ¬ K(x,y))
∀ x. ¬ L(x) ∨ (∀ y. A(y) → ¬ K(x,y))
∀ x. ¬ L(x) ∨ (∀ y. ¬ A(y) ∨ ¬ K(x,y))
¬ L(x) ∨ ¬ A(y) ∨ ¬ K(x,y)

Robinson's Resolution for FOL

Given KB = {P1(…), P2(…), ..., Pn(…)}. Prove Q(…).
Add ¬Q(…) to KB: KB = KB ∧ ¬Q(…). Prove unsatisfied.

Theorem: A set of clauses S is unsatisfiable if and only if upon the input S, Resolution procedure finds the empty clause (after a finite time).

1. Write each Pi(…), ¬Q(…) in one line.
2. Transfer to CNF representation

$\forall x_1 \forall x_2 \ldots \forall x_n\ [p_1(\ldots) \vee \ldots \vee p_n(\ldots)] \wedge [q_1(\ldots) \vee \ldots \vee q_m(\ldots)]$ (*)

3. Break (*) into smaller clauses at the logic connective ∧ :

~~$\forall x_1 \forall x_2 \ldots \forall x_n$~~ $[p_1(\ldots) \vee \ldots \vee p_n(\ldots)]$

~~$\forall x_1 \forall x_2 \ldots \forall x_n$~~ $[q_1(\ldots) \vee \ldots \vee q_m(\ldots)]$

Robinson's Resolution for FOL

4. Resolution:
   - u) $\neg p(x_1, x_2, \ldots, x_n) \vee q(\ldots)$
   - v) $p(y_1, y_2, \ldots, y_n) \vee r(\ldots)$

   with substitution $\overline{\ } = \left\{ \frac{z_1}{x_1}, \frac{z_1}{y_1}, \ldots, \frac{z_n}{x_n}, \frac{z_n}{y_n} \right\}$

5. Contrast appears when KB contains 2 lines:
   - i) $\neg p(x_1, x_2, \ldots, x_n)$
   - ii) $p(y_1, y_2, \ldots, y_n)$

   ⇒ w) q(…) ∨ r(…) with substitution

$$\overline{\ } = \left\{ \frac{z_1}{x_1}, \frac{z_1}{y_1}, \ldots, \frac{z_n}{x_n}, \frac{z_n}{y_n} \right\}$$

Step 4 - Example

- Which substitution θ is used for resolving:
  - P(a,x,b), and
  - ¬P(y,z,z)

$$\theta = \left\{\frac{a}{y}, \frac{b}{z}, \frac{b}{x}\right\}$$

- P(a,b,b)
- ¬P(a,b,b)

13

Step 4 - Example

$$\frac{\neg Pet(Joe) \vee Cat(Joe) \vee \overset{\downarrow}{Bird}(Joe) \qquad Parrot(x) \vee \neg \overset{\downarrow}{Bird}(x)}{\neg Pet(Joe) \vee Cat(Joe) \vee Parrot(Joe)} \quad (1)$$

(1) $mgu(Bird(x), Bird(Joe)) = \{x/Joe\}$

$$\frac{\neg \overset{\downarrow}{On}(x,y) \vee Above(x,y) \qquad On(B,A) \vee \overset{\downarrow}{On}(A,B)}{Above(A,B) \vee On(B,A)} \quad (2)$$

(2) $mgu(On(x,y), On(A,B)) = \{x/A,\ y/B\}$

$$\frac{\neg Bird(x) \vee \overset{\downarrow}{Feathers}(x) \qquad \neg \overset{\downarrow}{Feathers}(y) \vee Flies(y)}{\neg Bird(x) \vee Flies(x)} \quad (3)$$

(3) $mgu(Feathers(x), Feathers(y)) = \{y/x\}$

14

Step 4 - Example

- Which substitution θ is used for resolving:
  - P(a,x,x,b), and
  - ¬P(y,y,z,b)

15

Step 4 - Example

- Given p(a,b), p(c,d), q(d,c,c) are true
- Rule
  - p(x,y) ∧ q(y,x,x) ⇒ r(x,y)
- Using substitutions with the above rule, generate new predicate
- Idea:
  - Try with p(x,y) ≡ p(a,b) or p(x,y) ≡ p(c,d)

16

Resolution - Example

$\forall x \quad P(x) \to Q(x)$

$\forall x \quad \neg P(x) \to R(x)$

$\forall x \quad Q(x) \to S(x)$

$\forall x \quad R(x) \to S(x)$

Transform to CNF

1. $\neg P(x) \vee Q(x)$
2. $P(x) \vee R(x)$
3. $\neg Q(x) \vee S(x)$
4. $\neg R(x) \vee S(x)$

Resolve 1 and 3

$5. \neg P(x) \vee S(x)$

Resolve 2 and 5

$6. R(x) \vee S(x)$

Resolve 4 and 6

$7. S(x)$

Exercice 1

- John owns a dog
- Anyone who owns a dog is a lover of animals
- Lovers of animals do not kill animals
- Proves that John does not kill animals?

Transform the problem to a set of clauses and apply Robinson's resolution

∃x. D(x) ∧ O(John,x)	D(Fido) ∧ O(John,Fido)
∀x. (∃y.D(y)∧O(x,y)) → L(x)	¬D(y) ∨ ¬O(x,y) ∨ L(x)
∀x. L(x) → (∀y. A(y) → ¬K(x,y))	¬L(x) ∨ ¬A(y) ∨ ¬K(x,y)
∀x. D(x) ⇒ A(x)	¬D(x) ∨ A(x)
∀x. A(x) ⇒ ¬K(John,x)	A(Fido) ∧ K(John, Fido)

Exercice 2

- Jack owns a dog
- Every dog owner is an animal lover
- No animal lover kills an animal
- Either Jack or Curiosity killed the cat, who is named Tuna
- Prove that Curiosity kill the cat.

Jack owns a dog own(Jack, dog)
Every dog owner is an animal lover
No animal lover kills an animal
Either Jack or Curiosity killed the cat, who is named Tuna
Did Curiosity kill the cat? Kills(Curiosity,Tuna)

$\exists x. Dog(x) \wedge Owns(Jack, x)$

$\forall x \forall y. (Dog(y) \wedge Owns(x, y)) \Rightarrow AnimalLover(x)$

$\forall x. (\exists y. Dog(y) \wedge Owns(x, y)) \Rightarrow AnimalLover(x)$

$\forall x \forall y. (AnimalLover(x) \wedge Animal(y) \Rightarrow \neg Kills(x, y))$

$Kills(Jack, Tuna) \vee Kill(Curiosity, Tuna)$

$Cat(Tuna)$

$\forall x. Cat(x) \Rightarrow Animal(x)$

Transform the problem to set of clauses

$Dog(D)$

$Owns(Jack, D)$

$\neg Dog(y) \vee \neg Owns(x, y) \vee AnimalLover(x)$

$\neg AnimalLover(x) \wedge \neg Animal(y) \vee \neg Kills(x, y)$

$Kills(Jack, Tuna) \vee Kill(Curiosity, Tuna)$

$Cat(Tuna)$

$\neg Cat(x) \vee Animal(x)$

¬Kills(Curiosity,Tuna)

Exercice 3

- The law says that it is a crime for an American to sell weapons to hostile nations
- The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American
- Is West a criminal?

The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American

Modeling

"... it is a crime for an American to sell weapons to hostile nations":

$$\forall x, y, z \; American(x) \wedge Weapon(y) \wedge Nation(z) \wedge Hostile(z) \wedge Sells(x, z, y) \Rightarrow Criminal(x)$$

"Nono ... has some missiles":

$$\exists x \; Owns(Nono, x) \wedge Missile(x)$$

"All of its missiles were sold to it by Colonel West":

$$\forall x \; Owns(Nono, x) \wedge Missile(x) \Rightarrow Sells(West, Nono, x)$$

We will also need to know that missiles are weapons:

$$\forall x \; Missile(x) \Rightarrow Weapon(x)$$

and that an enemy of America counts as "hostile":

$$\forall x \; Enemy(x, America) \Rightarrow Hostile(x)$$

"West, who is American ...":

$American(West)$

"The country Nono ...":

$Nation(Nono)$

"Nono, an enemy of America ...":

$Enemy(Nono, America)$

$Nation(America)$

Transform the problem to set of clauses and Resolution