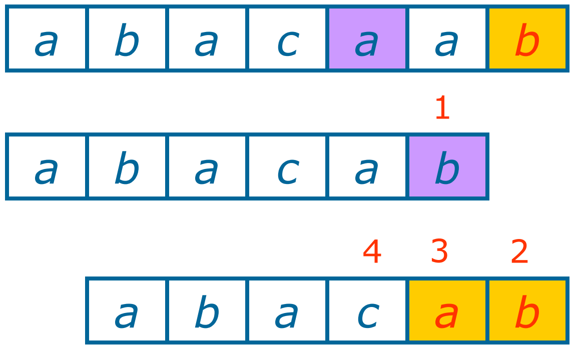
Data Structure & Algorithm Basic Lab – week 13

Topics of this week

- String pattern matching algorithms
  - Naive algorithm
  - Knuth-Morris-Pratt algorithm
  - Boyer-Moore algorithm
- Exercises



String matching problem

- Let P be a string of size m
  - A substring P[i .. j] of P is the subsequence of P consisting of the characters with ranks between i and j
  - A prefix of P is a substring of the type P[0 .. i]
  - A suffix of P is a substring of the type P[i ..m − 1]
- Given strings T (text) and P (pattern), the pattern matching problem consists of finding a substring of T equal to P
- Applications:
  - Text editors, Search engines, Biological research

Brute Force Matching

- The brute-force pattern matching algorithm compares the pattern P with the text T for each possible shift of P relative to T, until either
  - a match is found, or
  - all placements of the pattern have been tried
- Brute-force pattern matching runs in time O(nm)
- Example of worst case:
  - T = aaa … ah
  - P = aaah
  - may occur in images and DNA sequences
  - unlikely in English text

Algorithm

Algorithm BruteForceMatch(T, P)

```
// Input text T of size n and pattern P of size m
// Output starting index of a substring of T equal to P or
  -1
if no such substring exists
  for  i ← 0 to n − m {
     test shift i of the pattern
  }
  j ← 0
  while j < m ∧ T[i + j] = P[j]
     j ← j + 1
  if  j = m
     return  i {match at i}
  else
  break while loop {mismatch}
return  -1 {no match anywhere}
```

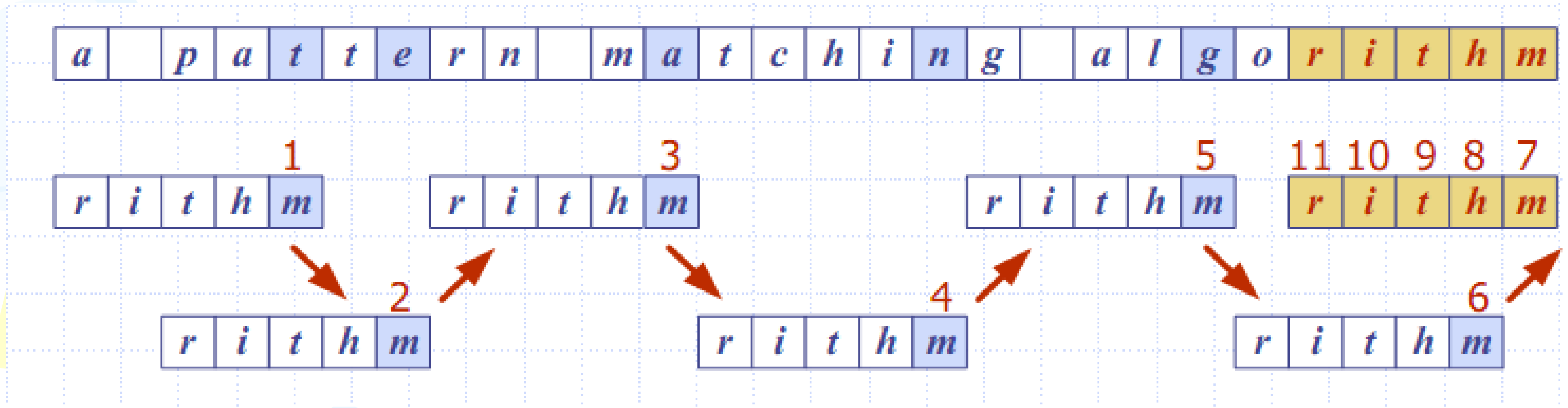
Exercise 13.1

- Make a random string that has about 2000 characters consisting of a set of characters..
- For example:
  - set of characters: abcdef
  - string: abcadacaeeeffaadbfacddedcedfbeccae…
- Write the program that searches the pattern, for example “aadbf”, from the string.
- Note: use Simple searching string Algorithm

Boyer-Moore Heuristics

- The Boyer-Moore's pattern matching algorithm is based on two heuristics
- Looking-glass heuristic: Compare P with a subsequence of T
- moving backwards
- Character-jump heuristic: When a mismatch occurs at T[i] = c
  - If P contains c, shift P to align the last occurrence of c in P with T[i]
  - Else, shift P to align P[0] with T[i + 1]

Example



Last-Occurrence Function

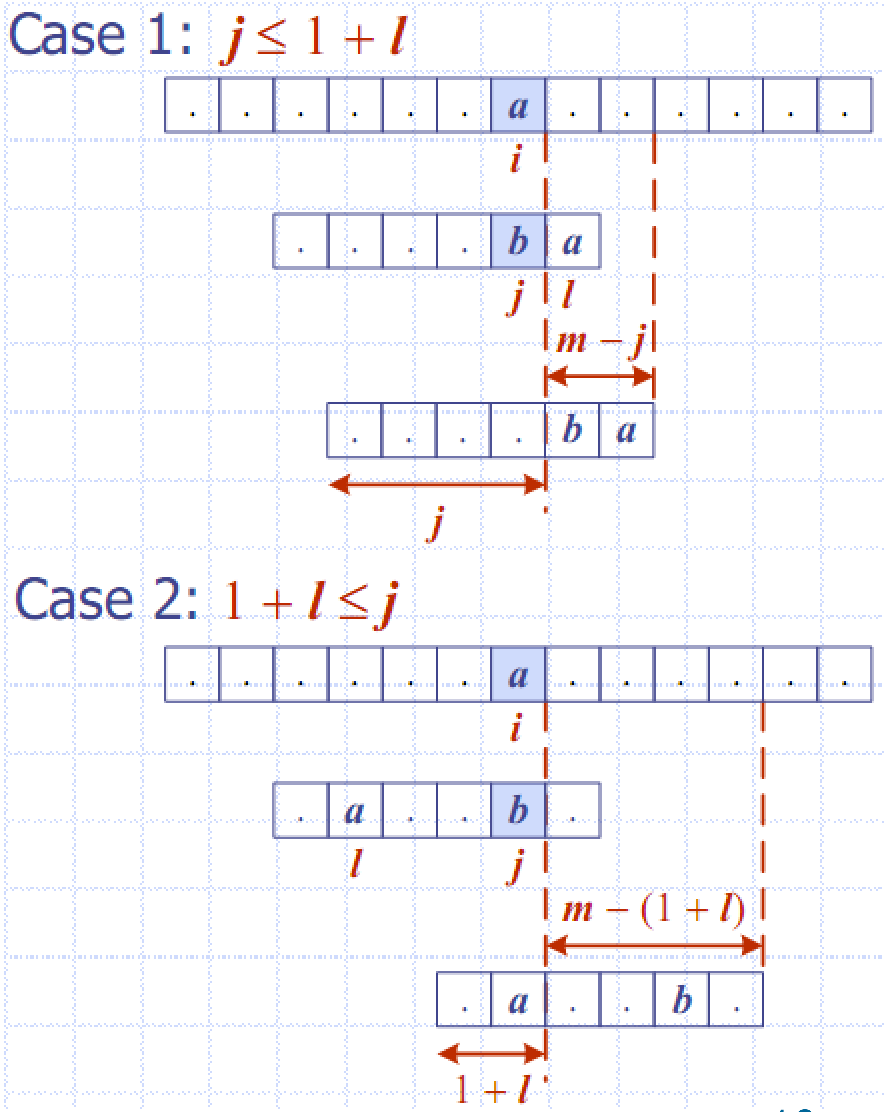
- Boyer-Moore's algorithm preprocesses the pattern P and the alphabet Σ to build the last-occurrence function L mapping Σ to integers, where L(c) is defined as
  - the largest index i such that P[i] = c or
  - −1 if no such index exists
- Example:
  - Σ = {a, b, c, d}
  - P = abacab

c	a	b	c	d
L(c)	4	5	3	-1

- The last-occurrence function can be represented by an array indexed by the numeric codes of the characters
- The last-occurrence function can be computed in time O(m + s), where m is the size of P and s is the size of Σ

Algorithm Boyer Moore

```
Algorithm BoyerMooreMatch(T, P, Σ)
    L ← lastOccurenceFunction(P, Σ )
    i ← m − 1
    j ← m − 1
    repeat
    if T[i] = P[j]
        if j = 0
        return  i  { match at i }
        else
        i ← i − 1
        j ← j − 1
    else
    { character-jump }
        l ← L[T[i]]
        i ← i + m – min(j, 1 + l)
        j ← m − 1
    until  i > n − 1
    return  −1 { no match }
```



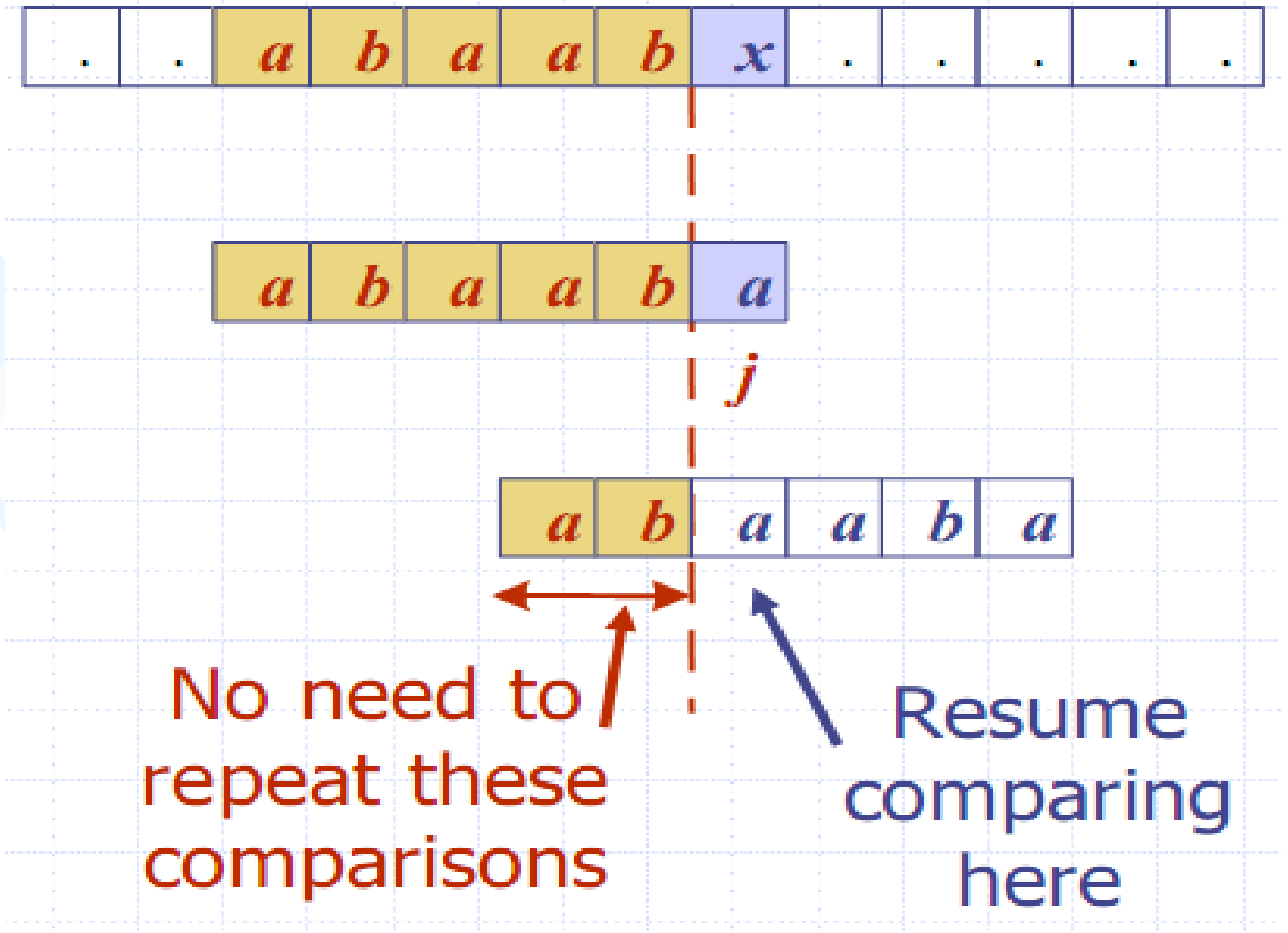
Exercise 13.2: Searching string by Boyer-Moore

- Make a random string that has about 2000 characters consisting of a set of characters.
- set of characters: abcdef
- string: abcadacaeeeffaadbfacddedcedfbeccae…
- Write the program that search the pattern, for example “aadbf”, from the string.
- Note: use Boyer-Moore Algorithm

KMP string matching

- Knuth-Morris-Pratt’s algorithm compares the pattern to the text in left-to-right, but shifts the pattern more intelligently than the brute-force algorithm.
- When a mismatch occurs, what is the most we can shift the pattern so as to avoid redundant comparisons?
- Answer: the largest prefix of P[0..j] that is a suffix of P[1..j]

Example



. . a b a a b x . . . . .
a b a a b a
j
a b a a b a
No need to repeat these comparisons
Resume comparing here

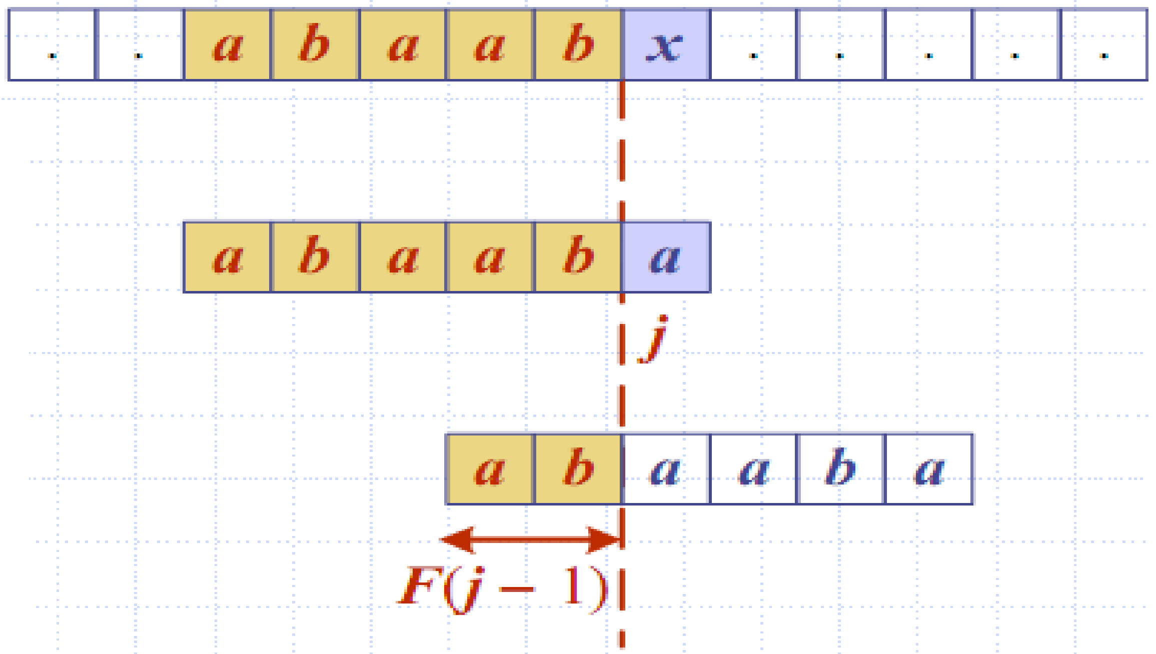
KMP Failure Function

- Knuth-Morris-Pratt's algorithm preprocesses the pattern to find matches of prefixes of the pattern with the pattern itself
- The failure function $F(j)$ is defined as the size of the largest prefix of $P[0..j]$ that is also a suffix of $P[1..j]$
- Knuth-Morris-Pratt's algorithm modifies the brute-force algorithm so that if a mismatch occurs at $P[j] \neq T[i]$ we set

$j \leftarrow F(j - 1)$

Example

j	0	1	2	3	4	5
$P[j]$	a	b	a	a	b	a
$F(j)$	0	0	1	1	2	3



```
Algorithm failureFunction(P)
   F[0] ← 0
   i ← 1
   j ← 0
   while i < m
      if P[i] = P[j]
      {we have matched j + 1 chars}
         F[i] ← j + 1
         i ← i + 1
         j ← j + 1
      else if  j > 0 then
      {use failure function to shift P}
         j ← F[j − 1]
      else
         F[i] ← 0 { no match }
         i ← i + 1
```

Exercise 13.3

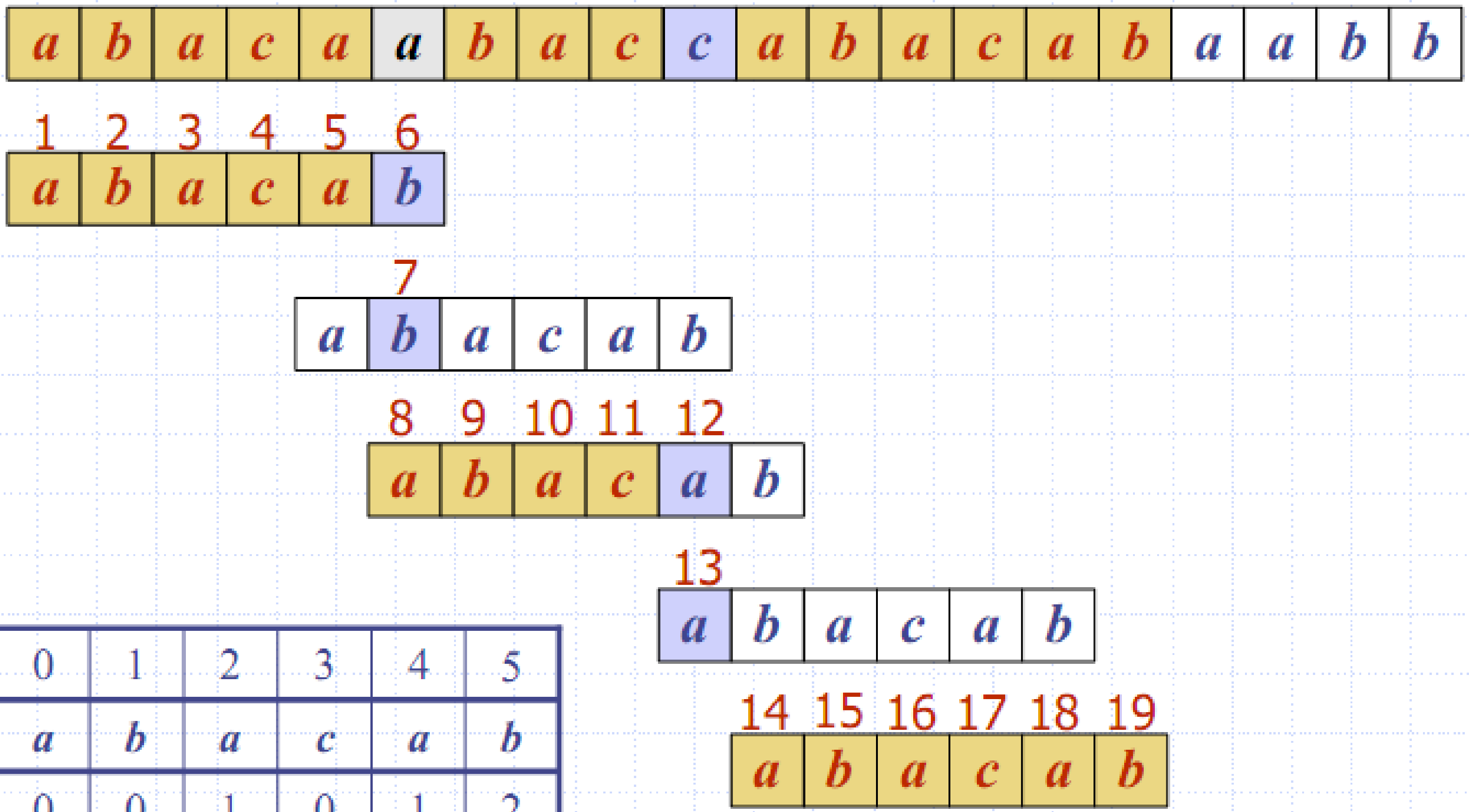
- Repeat the exercise 13.2 using the KMP algorithm.
- Calculate the number of comparisons.

The KMP algorithm

- The failure function can be represented by an array and can be computed in O(m) time
- At each iteration of the while-loop, either
  - i increases by one, or
  - the shift amount i – j increases by at least one (observe that F(j – 1) < j)
- Hence, there are no more than 2n iterations of the while-loop
- Thus, KMP's algorithm runs in optimal time O(m + n)

```
Algorithm KMPMatch(T, P)
    F ← failureFunction(P)
    i ← 0
    j ← 0
    while i < n
        if T[i] = P[j]
            if j = m − 1
                return i − j { match }
            else
                i ← i + 1
                j ← j + 1
        else
            if j > 0
                j ← F[j − 1]
            else
                i ← i + 1
    return −1 { no match }
```

Example



j	0	1	2	3	4	5
P[*j*]	*a*	*b*	*a*	*c*	*a*	*b*
F(*j*)	0	0	1	0	1	2